Summer Work for AP Calculus AB

1. Go to Larsonprecalculus.com and watch the videos for sections 9.1-9.2 on Sequences and Series, then do: p. 676: 1-55 odd

2. Go to Larsonprecalculus.com and watch the videos for 10.1-10.4 and 10.6-10.8 on Conics, Parametric Equations and Polar Coordinates, then do:

   p. 764: 1-43 odd, 53-103 odd

3. Go to Larsoncalculus.com and watch the videos for 7.3 on the Shell Method then do:

   p. 462: 1-35 odd

Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

9.1 Writing the Terms of a Sequence **In Exercises 1–4, write the first five terms of the sequence. (Assume that n begins with 1.)**

1. $a_n = 2 + \dfrac{6}{n}$
2. $a_n = \dfrac{(-1)^n 5n}{2n-1}$
3. $a_n = \dfrac{72}{n!}$
4. $a_n = n(n-1)$

Finding the nth Term of a Sequence **In Exercises 5–8, write an expression for the apparent nth term (a_n) of the sequence. (Assume that n begins with 1.)**

5. $-2, 2, -2, 2, -2, \ldots$
6. $-1, 2, 7, 14, 23, \ldots$
7. $4, 2, \frac{4}{3}, 1, \frac{4}{5}, \ldots$
8. $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \ldots$

Simplifying a Factorial Expression **In Exercises 9–12, simplify the factorial expression.**

9. $9!$
10. $4! \cdot 0!$
11. $\dfrac{3! \cdot 5!}{6!}$
12. $\dfrac{7! \cdot 6!}{6! \cdot 8!}$

Finding a Sum **In Exercises 13 and 14, find the sum.**

13. $\displaystyle\sum_{j=1}^{4} \frac{6}{j^2}$
14. $\displaystyle\sum_{k=1}^{10} 2k^3$

Using Sigma Notation to Write a Sum **In Exercises 15 and 16, use sigma notation to write the sum.**

15. $\dfrac{1}{2(1)} + \dfrac{1}{2(2)} + \dfrac{1}{2(3)} + \cdots + \dfrac{1}{2(20)}$
16. $\dfrac{1}{2} + \dfrac{2}{3} + \dfrac{3}{4} + \cdots + \dfrac{9}{10}$

Finding the Sum of an Infinite Series **In Exercises 17 and 18, find the sum of the infinite series.**

17. $\displaystyle\sum_{i=1}^{\infty} \frac{4}{10^i}$
18. $\displaystyle\sum_{k=1}^{\infty} 2\left(\frac{1}{100}\right)^k$

19. **Compound Interest** An investor deposits \$10,000 in an account that earns 2.25% interest compounded monthly. The balance in the account after n months is given by

$$A_n = 10{,}000\left(1 + \frac{0.0225}{12}\right)^n, \quad n = 1, 2, 3, \ldots$$

(a) Write the first 10 terms of the sequence.

(b) Find the balance in the account after 10 years by computing the 120th term of the sequence.

20. **Lottery Ticket Sales** The total sales a_n (in billions of dollars) of lottery tickets in the United States from 2001 through 2010 can be approximated by the model

$$a_n = -0.18n^2 + 3.7n + 35, \quad n = 1, 2, \ldots, 10$$

where n is the year, with $n = 1$ corresponding to 2001. Write the terms of this finite sequence. Use a graphing utility to construct a bar graph that represents the sequence. *(Source: TLF Publications, Inc.)*

9.2 Determining Whether a Sequence Is Arithmetic **In Exercises 21–24, determine whether the sequence is arithmetic. If so, then find the common difference.**

21. $6, -1, -8, -15, -22, \ldots$
22. $0, 1, 3, 6, 10, \ldots$
23. $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$
24. $1, \frac{15}{16}, \frac{7}{8}, \frac{13}{16}, \frac{3}{4}, \ldots$

Finding the nth Term **In Exercises 25–28, find a formula for a_n for the arithmetic sequence.**

25. $a_1 = 7, d = 12$
26. $a_1 = 28, d = -5$
27. $a_2 = 93, a_6 = 65$
28. $a_7 = 8, a_{13} = 6$

Writing the Terms of an Arithmetic Sequence **In Exercises 29 and 30, write the first five terms of the arithmetic sequence.**

29. $a_1 = 3, d = 11$
30. $a_1 = 25, a_{k+1} = a_k + 3$

31. **Sum of a Finite Arithmetic Sequence** Find the sum of the first 100 positive multiples of 7.

32. **Sum of a Finite Arithmetic Sequence** Find the sum of the integers from 40 to 90 (inclusive).

Finding a Partial Sum **In Exercises 33–36, find the partial sum.**

33. $\displaystyle\sum_{j=1}^{10} (2j - 3)$
34. $\displaystyle\sum_{j=1}^{8} (20 - 3j)$
35. $\displaystyle\sum_{k=1}^{11} \left(\frac{2}{3}k + 4\right)$
36. $\displaystyle\sum_{k=1}^{25} \left(\frac{3k+1}{4}\right)$

37. **Job Offer** The starting salary for a job is \$43,800 with a guaranteed increase of \$1950 per year. Determine (a) the salary during the fifth year and (b) the total compensation through five full years of employment.

38. **Baling Hay** In the first two trips baling hay around a large field, a farmer obtains 123 bales and 112 bales, respectively. Because each round gets shorter, the farmer estimates that the same pattern will continue. Estimate the total number of bales made after the farmer takes another six trips around the field.

9.3 Determining Whether a Sequence Is Geometric **In Exercises 39–42, determine whether the sequence is geometric. If so, then find the common ratio.**

39. $6, 12, 24, 48, \ldots$

40. $54, -18, 6, -2, \ldots$

41. $\frac{1}{5}, -\frac{3}{5}, \frac{9}{5}, -\frac{27}{5}, \ldots$

42. $\frac{1}{4}, \frac{2}{5}, \frac{3}{6}, \frac{4}{7}, \ldots$

Writing the Terms of a Geometric Sequence **In Exercises 43–46, write the first five terms of the geometric sequence.**

43. $a_1 = 2,\ r = 15$

44. $a_1 = 4,\ r = -\frac{1}{4}$

45. $a_1 = 9,\ a_3 = 4$

46. $a_1 = 2,\ a_3 = 12$

Finding a Term of a Geometric Sequence **In Exercises 47–50, write an expression for the *n*th term of the geometric sequence. Then find the 10th term of the sequence.**

47. $a_1 = 18, a_2 = -9$

48. $a_3 = 6, a_4 = 1$

49. $a_1 = 100, r = 1.05$

50. $a_1 = 5, r = 0.2$

Sum of a Finite Geometric Sequence **In Exercises 51–56, find the sum of the finite geometric sequence.**

51. $\sum_{i=1}^{7} 2^{i-1}$

52. $\sum_{i=1}^{5} 3^{i-1}$

53. $\sum_{i=1}^{4} \left(\frac{1}{2}\right)^i$

54. $\sum_{i=1}^{6} \left(\frac{1}{3}\right)^{i-1}$

55. $\sum_{i=1}^{5} (2)^{i-1}$

56. $\sum_{i=1}^{4} 6(3)^i$

Sum of a Finite Geometric Sequence **In Exercises 57 and 58, use a graphing utility to find the sum of the finite geometric sequence.**

57. $\sum_{i=1}^{10} 10\left(\frac{3}{5}\right)^{i-1}$

58. $\sum_{i=1}^{15} 20(0.2)^{i-1}$

Sum of an Infinite Geometric Sequence **In Exercises 59–62, find the sum of the infinite geometric series.**

59. $\sum_{i=0}^{\infty} \left(\frac{7}{8}\right)^i$

60. $\sum_{i=0}^{\infty} (0.5)^i$

61. $\sum_{k=1}^{\infty} 4\left(\frac{2}{3}\right)^{k-1}$

62. $\sum_{k=1}^{\infty} 1.3\left(\frac{1}{10}\right)^{k-1}$

63. Depreciation A paper manufacturer buys a machine for \$120,000. During the next 5 years, it will depreciate at a rate of 30% per year. (In other words, at the end of each year the depreciated value will be 70% of what it was at the beginning of the year.)

(a) Find the formula for the *n*th term of a geometric sequence that gives the value of the machine *t* full years after it was purchased.

(b) Find the depreciated value of the machine after 5 full years.

64. Annuity An investor deposits \$800 in an account on the first day of each month for 10 years. The account pays 3%, compounded monthly. What will the balance be at the end of 10 years?

9.4 Using Mathematical Induction **In Exercises 65–68, use mathematical induction to prove the formula for every positive integer *n*.**

65. $3 + 5 + 7 + \cdots + (2n + 1) = n(n + 2)$

66. $1 + \frac{3}{2} + 2 + \frac{5}{2} + \cdots + \frac{1}{2}(n + 1) = \frac{n}{4}(n + 3)$

67. $\sum_{i=0}^{n-1} ar^i = \frac{a(1 - r^n)}{1 - r}$

68. $\sum_{k=0}^{n-1} (a + kd) = \frac{n}{2}[2a + (n - 1)d]$

Finding a Formula for a Sum **In Exercises 69–72, use mathematical induction to find a formula for the sum of the first *n* terms of the sequence.**

69. $9, 13, 17, 21, \ldots$

70. $68, 60, 52, 44, \ldots$

71. $1, \frac{3}{5}, \frac{9}{25}, \frac{27}{125}, \ldots$

72. $12, -1, \frac{1}{12}, -\frac{1}{144}, \ldots$

Finding a Sum **In Exercises 73 and 74, find the sum using the formulas for the sums of powers of integers.**

73. $\sum_{n=1}^{75} n$

74. $\sum_{n=1}^{6} (n^5 - n^2)$

Linear Model, Quadratic Model, or Neither? **In Exercises 75 and 76, write the first five terms of the sequence beginning with the given term. Then calculate the first and second differences of the sequence. State whether the sequence has a perfect linear model, a perfect quadratic model, or neither.**

75. $a_1 = 5$
$a_n = a_{n-1} + 5$

76. $a_1 = -3$
$a_n = a_{n-1} - 2n$

9.5 Finding a Binomial Coefficient **In Exercises 77 and 78, find the binomial coefficient.**

77. ${}_6C_4$

78. ${}_{12}C_3$

Using Pascal's Triangle **In Exercises 79 and 80, evaluate using Pascal's Triangle.**

79. $\binom{7}{2}$

80. $\binom{10}{4}$

Expanding an Expression **In Exercises 81–84, use the Binomial Theorem to expand and simplify the expression. (Remember that $i = \sqrt{-1}$.)**

81. $(x + 4)^4$

82. $(a - 3b)^5$

83. $(5 + 2i)^4$

84. $(4 - 5i)^3$

1. 8, 5, 4, $\frac{7}{2}$, $\frac{16}{5}$ **3.** 72, 36, 12, 3, $\frac{3}{5}$ **5.** $a_n = 2(-1)^n$

7. $a_n = \dfrac{4}{n}$ **9.** 362,880 **11.** 1 **13.** $\dfrac{205}{24}$

15. $\displaystyle\sum_{k=1}^{20} \frac{1}{2k}$ **17.** $\dfrac{4}{9}$

19. (a) $A_1 = \$10{,}018.75$
$A_2 \approx \$10{,}037.54$
$A_3 \approx \$10{,}056.36$
$A_4 \approx \$10{,}075.21$
$A_5 \approx \$10{,}094.10$
$A_6 \approx \$10{,}113.03$
$A_7 \approx \$10{,}131.99$
$A_8 \approx \$10{,}150.99$
$A_9 \approx \$10{,}170.02$
$A_{10} \approx \$10{,}189.09$
(b) \$12,520.59

21. Arithmetic sequence, $d = -7$

23. Arithmetic sequence, $d = \frac{1}{2}$ **25.** $a_n = 12n - 5$

27. $a_n = -7n + 107$ **29.** 3, 14, 25, 36, 47 **31.** 35,350

33. 80 **35.** 88 **37.** (a) \$51,600 (b) \$238,500

39. Geometric sequence, $r = 2$

41. Geometric sequence, $r = -3$

43. 2, 30, 450, 6750, 101,250

45. 9, 6, 4, $\frac{8}{3}$, $\frac{16}{9}$ or 9, -6, 4, $-\frac{8}{3}$, $\frac{16}{9}$ **47.** $a_n = 18\left(-\frac{1}{2}\right)^{n-1}$; $\frac{9}{512}$

49. $a_n = 100(1.05)^{n-1}$; About 155.133 **51.** 127 **53.** $\frac{15}{16}$

55. 31 **57.** 24.85 **59.** 8 **61.** 12

Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

10.1 Finding the Inclination of a Line **In Exercises 1–4, find the inclination θ (in radians and degrees) of the line with the given characteristics.**

1. Passes through the points $(-1, 2)$ and $(2, 5)$
2. Passes through the points $(3, 4)$ and $(-2, 7)$
3. Equation: $y = 2x + 4$
4. Equation: $x - 5y = 7$

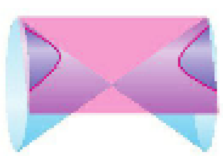
Finding the Angle Between Two Lines **In Exercises 5–8, find the angle θ (in radians and degrees) between the lines.**

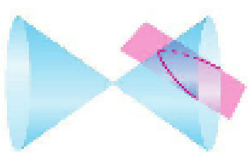
5. $4x + y = 2$, $-5x + y = -1$
6. $-5x + 3y = 3$, $-2x + 3y = 1$
7. $2x - 7y = 8$, $\frac{2}{5}x + y = 0$
8. $0.02x + 0.07y = 0.18$, $0.09x - 0.04y = 0.17$

Finding the Distance Between a Point and a Line **In Exercises 9 and 10, find the distance between the point and the line.**

	Point	Line
9.	$(5, 3)$	$x - y = 10$
10.	$(0, 4)$	$x + 2y = 2$

10.2 Forming a Conic Section **In Exercises 11 and 12, state what type of conic is formed by the intersection of the plane and the double-napped cone.**

11. 

12. 

Finding the Standard Equation of a Parabola **In Exercises 13–16, find the standard form of the equation of the parabola with the given characteristics. Then sketch the parabola.**

13. Vertex: $(0, 0)$; Focus: $(4, 0)$
14. Vertex: $(2, 0)$; Focus: $(0, 0)$
15. Vertex: $(0, 2)$; Directrix: $x = -3$
16. Vertex: $(-3, -3)$; Directrix: $y = 0$

Finding the Tangent Line at a Point on a Parabola **In Exercises 17 and 18, find the equation of the tangent line to the parabola at the given point.**

17. $y = 2x^2$, $(-1, 2)$
18. $x^2 = -2y$, $(-4, -8)$

19. **Architecture** A parabolic archway is 10 meters high at the vertex. At a height of 8 meters, the width of the archway is 6 meters (see figure). How wide is the archway at ground level?

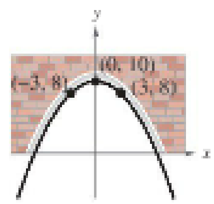


Figure for 19

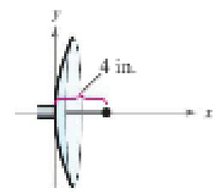


Figure for 20

20. **Parabolic Microphone** The receiver of a parabolic microphone is at the focus of its parabolic reflector, 4 inches from the vertex (see figure). Write an equation of a cross section of the reflector with its focus on the positive x-axis and its vertex at the origin.

10.3 Finding the Standard Equation of an Ellipse **In Exercises 21–24, find the standard form of the equation of the ellipse with the given characteristics.**

21. Vertices: $(-2, 0)$, $(8, 0)$; foci: $(0, 0)$, $(6, 0)$
22. Vertices: $(4, 3)$, $(4, 7)$; foci: $(4, 4)$, $(4, 6)$
23. Vertices: $(0, 1)$, $(4, 1)$; endpoints of the minor axis: $(2, 0)$, $(2, 2)$
24. Vertices: $(-4, -1)$, $(-4, 11)$; endpoints of the minor axis: $(-6, 5)$, $(-2, 5)$

25. **Architecture** A contractor plans to construct a semielliptical arch 10 feet wide and 4 feet high. Where should the foci be placed in order to sketch the arch?

26. **Wading Pool** You are building a wading pool that is in the shape of an ellipse. Your plans give an equation for the elliptical shape of the pool measured in feet as

$$\frac{x^2}{324} + \frac{y^2}{196} = 1.$$

Find the longest distance across the pool, the shortest distance, and the distance between the foci.

Sketching an Ellipse **In Exercises 27–30, find the center, vertices, foci, and eccentricity of the ellipse. Then sketch the ellipse.**

27. $\dfrac{(x + 1)^2}{25} + \dfrac{(y - 2)^2}{49} = 1$
28. $\dfrac{(x - 5)^2}{1} + \dfrac{(y + 3)^2}{36} = 1$
29. $16x^2 + 9y^2 - 32x + 72y + 16 = 0$
30. $4x^2 + 25y^2 + 16x - 150y + 141 = 0$

10.4 **Finding the Standard Equation of a Hyperbola** **In Exercises 31–34, find the standard form of the equation of the hyperbola with the given characteristics.**

31. Vertices: $(0, \pm 1)$; foci: $(0, \pm 2)$
32. Vertices: $(3, 3), (-3, 3)$; foci: $(4, 3), (-4, 3)$
33. Foci: $(\pm 5, 0)$; asymptotes: $y = \pm \frac{3}{4}x$
34. Foci: $(0, \pm 13)$; asymptotes: $y = \pm \frac{5}{12}x$

Sketching a Hyperbola **In Exercises 35–38, find the center, vertices, foci, and the equations of the asymptotes of the hyperbola. Then sketch the hyperbola using the asymptotes as an aid.**

35. $\dfrac{(x-5)^2}{36} - \dfrac{(y+3)^2}{16} = 1$
36. $\dfrac{(y-1)^2}{4} - x^2 = 1$
37. $9x^2 - 16y^2 - 18x - 32y - 151 = 0$
38. $-4x^2 + 25y^2 - 8x + 150y + 121 = 0$

39. **Navigation** Radio transmitting station A is located 200 miles east of transmitting station B. A ship is in an area to the north and 40 miles west of station A. Synchronized radio pulses transmitted at 186,000 miles per second by the two stations are received 0.0005 second sooner from station A than from station B. How far north is the ship?

40. **Locating an Explosion** Two of your friends live 4 miles apart and on the same "east-west" street, and you live halfway between them. You are having a three-way phone conversation when you hear an explosion. Six seconds later, your friend to the east hears the explosion, and your friend to the west hears it 8 seconds after you do. Find equations of two hyperbolas that would locate the explosion. (Assume that the coordinate system is measured in feet and that sound travels at 1100 feet per second.)

Classifying a Conic from a General Equation **In Exercises 41–44, classify the graph of the equation as a circle, a parabola, an ellipse, or a hyperbola.**

41. $5x^2 - 2y^2 + 10x - 4y + 17 = 0$
42. $-4y^2 + 5x + 3y + 7 = 0$
43. $3x^2 + 2y^2 - 12x + 12y + 29 = 0$
44. $4x^2 + 4y^2 - 4x + 8y - 11 = 0$

10.5 **Rotation of Axes** **In Exercises 45–48, rotate the axes to eliminate the xy-term in the equation. Then write the equation in standard form. Sketch the graph of the resulting equation, showing both sets of axes.**

45. $xy + 3 = 0$
46. $x^2 - 4xy + y^2 + 9 = 0$
47. $5x^2 - 2xy + 5y^2 - 12 = 0$
48. $4x^2 + 8xy + 4y^2 + 7\sqrt{2}x + 9\sqrt{2}y = 0$

Rotation and Graphing Utilities **In Exercises 49–52, (a) use the discriminant to classify the graph of the equation, (b) use the Quadratic Formula to solve for y, and (c) use a graphing utility to graph the equation.**

49. $16x^2 - 24xy + 9y^2 - 30x - 40y = 0$
50. $13x^2 - 8xy + 7y^2 - 45 = 0$
51. $x^2 + y^2 + 2xy + 2\sqrt{2}x - 2\sqrt{2}y + 2 = 0$
52. $x^2 - 10xy + y^2 + 1 = 0$

10.6 **Sketching a Curve** **In Exercises 53 and 54, (a) create a table of x- and y-values for the parametric equations using $t = -2, -1, 0, 1,$ and 2, and (b) plot the points (x, y) generated in part (a) and sketch a graph of the parametric equations.**

53. $x = 3t - 2$ and $y = 7 - 4t$
54. $x = \dfrac{1}{4}t$ and $y = \dfrac{6}{t+3}$

Sketching a Curve **In Exercises 55–60, (a) sketch the curve represented by the parametric equations (indicate the orientation of the curve) and (b) eliminate the parameter and write the resulting rectangular equation whose graph represents the curve. Adjust the domain of the rectangular equation, if necessary. (c) Verify your result with a graphing utility.**

55. $x = 2t$, $y = 4t$
56. $x = 1 + 4t$, $y = 2 - 3t$
57. $x = t^2$, $y = \sqrt{t}$
58. $x = t + 4$, $y = t^2$
59. $x = 3\cos\theta$, $y = 3\sin\theta$
60. $x = 3 + 3\cos\theta$, $y = 2 + 5\sin\theta$

Finding Parametric Equations for a Graph **In Exercises 61–64, find a set of parametric equations to represent the graph of the rectangular equation using (a) $t = x$, (b) $t = x + 1$, and (c) $t = 3 - x$.**

61. $y = 2x + 3$
62. $y = 4 - 3x$
63. $y = x^2 + 3$
64. $y = 2 - x^2$
65. $y = 2x^2 + 2$
66. $y = 1 - 4x^2$

10.7 **Plotting Points in the Polar Coordinate System** **In Exercises 67–70, plot the point given in polar coordinates and find two additional polar representations of the point, using $-2\pi < \theta < 2\pi$.**

67. $\left(2, \dfrac{\pi}{4}\right)$
68. $\left(-5, -\dfrac{\pi}{3}\right)$
69. $(-7, 4.19)$
70. $\left(\sqrt{3}, 2.62\right)$

Polar-to-Rectangular Conversion **In Exercises 71–74, a point in polar coordinates is given. Convert the point to rectangular coordinates.**

71. $\left(-1, \frac{\pi}{3}\right)$ **72.** $\left(2, \frac{5\pi}{4}\right)$

73. $\left(3, \frac{3\pi}{4}\right)$ **74.** $\left(0, \frac{\pi}{2}\right)$

Rectangular-to-Polar Conversion **In Exercises 75–78, a point in rectangular coordinates is given. Convert the point to polar coordinates.**

75. $(0, 1)$ **76.** $(-\sqrt{5}, \sqrt{5})$

77. $(4, 6)$ **78.** $(3, -4)$

Converting a Rectangular Equation to Polar Form **In Exercises 79–84, convert the rectangular equation to polar form.**

79. $x^2 + y^2 = 81$ **80.** $x^2 + y^2 = 48$

81. $x^2 + y^2 - 6y = 0$ **82.** $x^2 + y^2 - 4x = 0$

83. $xy = 5$ **84.** $xy = -2$

Converting a Polar Equation to Rectangular Form **In Exercises 85–90, convert the polar equation to rectangular form.**

85. $r = 5$ **86.** $r = 12$

87. $r = 3\cos\theta$ **88.** $r = 8\sin\theta$

89. $r^2 = \sin\theta$ **90.** $r^2 = 4\cos 2\theta$

10.8 Sketching the Graph of a Polar Equation **In Exercises 91–100, sketch the graph of the polar equation using symmetry, zeros, maximum *r*-values, and any other additional points.**

91. $r = 6$ **92.** $r = 11$

93. $r = 4\sin 2\theta$ **94.** $r = \cos 5\theta$

95. $r = -2(1 + \cos\theta)$ **96.** $r = 1 - 4\cos\theta$

97. $r = 2 + 6\sin\theta$ **98.** $r = 5 - 5\cos\theta$

99. $r = -3\cos 2\theta$ **100.** $r^2 = \cos 2\theta$

Identifying Types of Polar Graphs **In Exercises 101–104, identify the type of polar graph and use a graphing utility to graph the equation.**

101. $r = 3(2 - \cos\theta)$ **102.** $r = 5(1 - 2\cos\theta)$

103. $r = 8\cos 3\theta$ **104.** $r^2 = 2\sin 2\theta$

10.9 Sketching a Conic **In Exercises 105–108, identify the conic and sketch its graph.**

105. $r = \dfrac{1}{1 + 2\sin\theta}$ **106.** $r = \dfrac{6}{1 + \sin\theta}$

107. $r = \dfrac{4}{5 - 3\cos\theta}$ **108.** $r = \dfrac{16}{4 + 5\cos\theta}$

Finding the Polar Equation of a Conic **In Exercises 109–112, find a polar equation of the conic with its focus at the pole.**

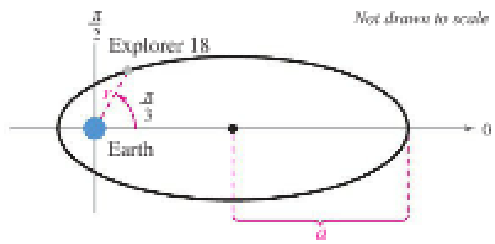
109. Parabola Vertex: $(2, \pi)$

110. Parabola Vertex: $(2, \pi/2)$

111. Ellipse Vertices: $(5, 0), (1, \pi)$

112. Hyperbola Vertices: $(1, 0), (7, 0)$

113. Explorer 18 On November 27, 1963, the United States launched Explorer 18. Its low and high points above the surface of Earth were 110 miles and 122,800 miles, respectively. The center of Earth was at one focus of the orbit (see figure). Find the polar equation of the orbit and find the distance between the surface of Earth (assume Earth has a radius of 4000 miles) and the satellite when $\theta = \pi/3$.



114. Asteroid An asteroid takes a parabolic path with Earth as its focus. It is about 6,000,000 miles from Earth at its closest approach. Write the polar equation of the path of the asteroid with its vertex at $\theta = \pi/2$. Find the distance between the asteroid and Earth when $\theta = -\pi/3$.

Exploration

True or False? **In Exercises 115–117, determine whether the statement is true or false. Justify your answer.**

115. The graph of $\frac{1}{4}x^2 - y^4 = 1$ is a hyperbola.

116. Only one set of parametric equations can represent the line $y = 3 - 2x$.

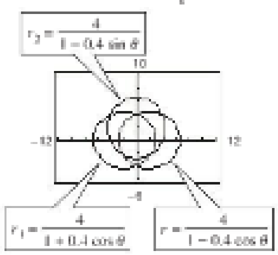
117. There is a unique polar coordinate representation of each point in the plane.

118. Think About It Consider an ellipse with the major axis horizontal and 10 units in length. The number b in the standard form of the equation of the ellipse must be less than what real number? Explain the change in the shape of the ellipse as b approaches this number.

119. Think About It What is the relationship between the graphs of the rectangular and polar equations?

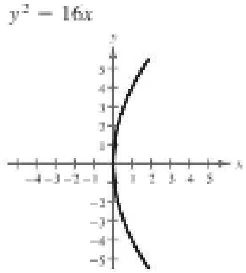
(a) $x^2 + y^2 = 25,\quad r = 5$

(b) $x - y = 0,\quad \theta = \dfrac{\pi}{4}$

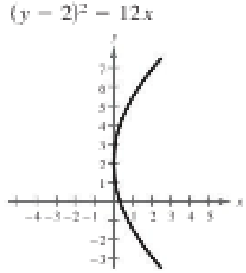
79. (a) Ellipse
(b) The given polar equation, r, has a vertical directrix to the left of the pole. The equation r_1 has a vertical directrix to the right of the pole, and the equation r_2 has a horizontal directrix below the pole.
(c)


Review Exercises *(page 764)*

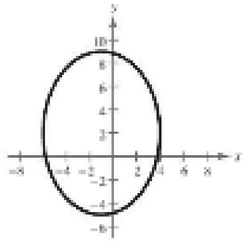
1. $\frac{\pi}{4}$ rad, 45° **3.** 1.1071 rad, 63.43° **5.** 0.4424 rad, 25.35°
7. 0.6588 rad, 37.75° **9.** $4\sqrt{2}$ **11.** Hyperbola
13. $y^2 = 16x$



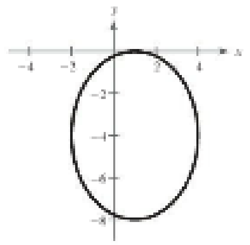
15. $(y - 2)^2 = 12x$



17. $y = -4x - 2$ **19.** $6\sqrt{5}$ m
21. $\frac{(x-3)^2}{25} + \frac{y^2}{16} = 1$ **23.** $\frac{(x-2)^2}{4} + \frac{(y-1)^2}{1} = 1$
25. The foci occur 3 feet from the center of the arch.
27. Center: $(-1, 2)$
Vertices:
$(-1, 9), (-1, -5)$
Foci: $\left(-1, 2 \pm 2\sqrt{6}\right)$
Eccentricity: $\frac{2\sqrt{6}}{7}$

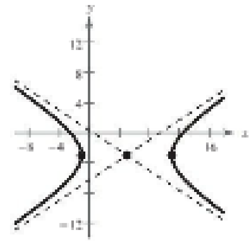


29. Center: $(1, -4)$
Vertices:
$(1, 0), (1, -8)$
Foci: $\left(1, -4 \pm \sqrt{7}\right)$
Eccentricity: $\frac{\sqrt{7}}{4}$

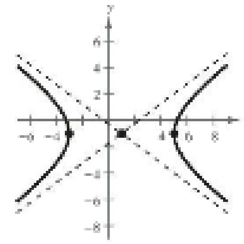


31. $\frac{y^2}{1} - \frac{x^2}{3} = 1$ **33.** $\frac{x^2}{16} - \frac{y^2}{9} = 1$

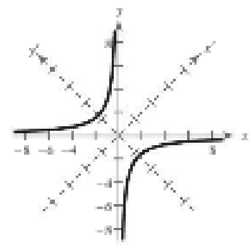
35. Center: $(5, -3)$
Vertices:
$(11, -3), (-1, -3)$
Foci: $\left(5 \pm 2\sqrt{13}, -3\right)$
Asymptotes:
$y = -3 \pm \frac{2}{3}(x - 5)$



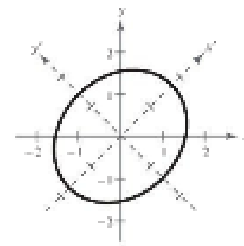
37. Center: $(1, -1)$
Vertices:
$(5, -1), (-3, -1)$
Foci: $(6, -1), (-4, -1)$
Asymptotes:
$y = -1 \pm \frac{3}{4}(x - 1)$

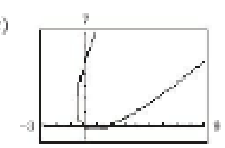


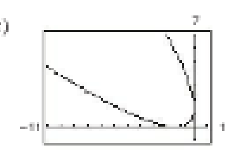
39. 72 mi **41.** Hyperbola **43.** Ellipse
45. $\frac{(y')^2}{6} - \frac{(x')^2}{6} = 1$



47. $\frac{(x')^2}{3} + \frac{(y')^2}{2} = 1$

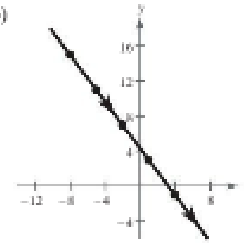


49. (a) Parabola
(b) $y = \frac{24x + 40 \pm \sqrt{(24x + 40)^2 - 36(16x^2 - 30x)}}{18}$
(c)


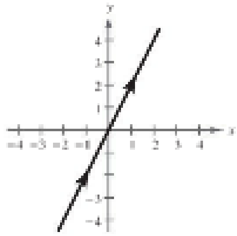
51. (a) Parabola
(b) $y = \frac{-\left(2x - 2\sqrt{2}\right) \pm \sqrt{\left(2x - 2\sqrt{2}\right)^2 - 4\left(x^2 + 2\sqrt{2x} + 2\right)}}{2}$
(c)


53. (a)

t	-2	-1	0	1	2
x	-8	-5	-2	1	4
y	15	11	7	3	-1

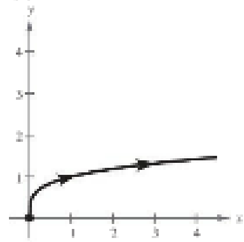
(b)


55. (a)



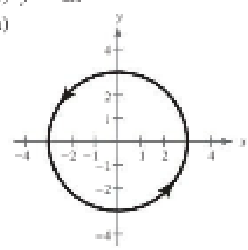
(b) $y = 2x$

57. (a)



(b) $y = \sqrt[4]{x}$

59. (a)



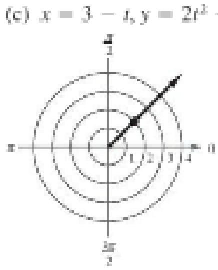
(b) $x^2 + y^2 = 9$

61. (a) $x = t,\ y = 2t + 3$ (b) $x = t - 1,\ y = 2t + 1$
(c) $x = 3 - t,\ y = 9 - 2t$

63. (a) $x = t,\ y = t^2 + 3$ (b) $x = t - 1,\ y = t^2 - 2t + 4$
(c) $x = 3 - t,\ y = t^2 - 6t + 12$

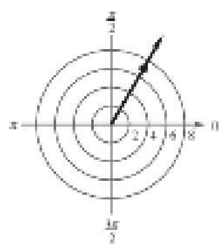
65. (a) $x = t,\ y = 2t^2 + 2$ (b) $x = t - 1,\ y = 2t^2 - 4t + 4$
(c) $x = 3 - t,\ y = 2t^2 - 12t + 20$

67.



$\left(2, -\frac{7\pi}{4}\right), \left(-2, \frac{5\pi}{4}\right)$

69.



$(7, 1.05), (-7, -2.09)$

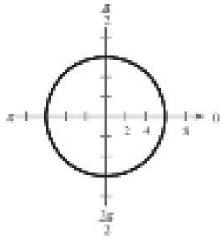
71. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ **73.** $\left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$ **75.** $\left(1, \frac{\pi}{2}\right)$

77. $\left(2\sqrt{13}, 0.9828\right)$ **79.** $r = 9$ **81.** $r = 6 \sin \theta$

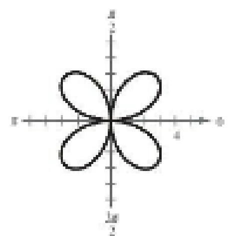
83. $r^2 = 10 \csc 2\theta$ **85.** $x^2 + y^2 = 25$

87. $x^2 + y^2 = 3x$ **89.** $x^2 + y^2 = y^{2/3}$

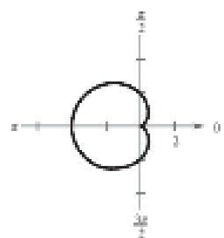
91. Symmetry: $\theta = \frac{\pi}{2}$, polar axis, pole
Maximum value of $|r|$: $|r| = 6$ for all values of θ
No zeros of r



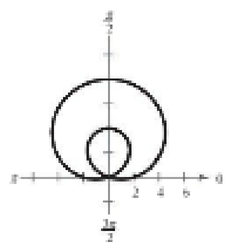
93. Symmetry: $\theta = \frac{\pi}{2}$, polar axis, pole
Maximum value of $|r|$: $|r| = 4$ when $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
Zeros of r: $r = 0$ when $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$



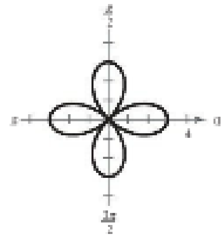
95. Symmetry: polar axis
Maximum value of $|r|$: $|r| = 4$ when $\theta = 0$
Zeros of r: $r = 0$ when $\theta = \pi$



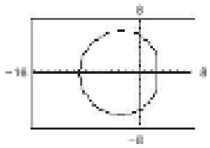
97. Symmetry: $\theta = \frac{\pi}{2}$
Maximum value of $|r|$: $|r| = 8$ when $\theta = \frac{\pi}{2}$
Zeros of r: $r = 0$ when $\theta = 3.4814, 5.9433$



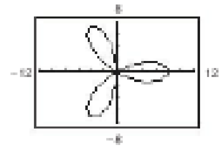
99. Symmetry: $\theta = \frac{\pi}{2}$, polar axis, pole
Maximum value of $|r|$: $|r| = 3$ when $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$
Zeros of r: $r = 0$ when $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$



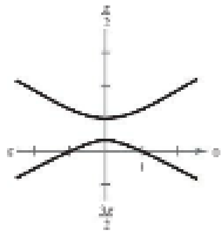
101. Limaçon



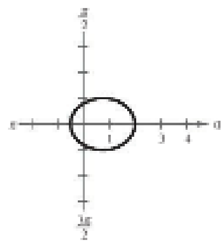
103. Rose curve



105. Hyperbola



107. Ellipse



109. $r = \dfrac{4}{1 - \cos\theta}$ **111.** $r = \dfrac{5}{3 - 2\cos\theta}$

113. $r = \dfrac{7961.93}{1 - 0.937\cos\theta}$; 10,980.11 mi

115. False. The equation of a hyperbola is a second-degree equation.

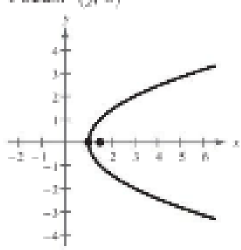
117. False. $(2, \pi/4)$, $(-2, 5\pi/4)$, and $(2, 9\pi/4)$ all represent the same point.

119. (a) The graphs are the same. (b) The graphs are the same.

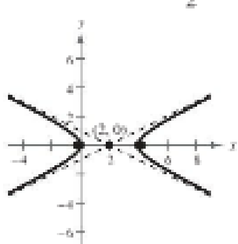
Chapter Test *(page 767)*

1. 0.3805 rad, 21.8° **2.** 0.8330 rad, 47.7° **3.** $\dfrac{7\sqrt{2}}{2}$

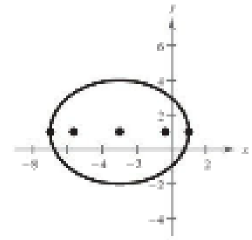
4. Parabola: $y^2 = 2(x - 1)$
Vertex: $(1, 0)$
Focus: $\left(\frac{3}{2}, 0\right)$



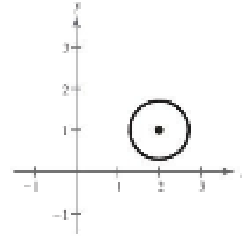
5. Hyperbola: $\dfrac{(x-2)^2}{4} - y^2 = 1$
Center: $(2, 0)$
Vertices: $(0, 0)$, $(4, 0)$
Foci: $\left(2 \pm \sqrt{5}, 0\right)$
Asymptotes: $y = \pm\dfrac{1}{2}(x - 2)$



6. Ellipse: $\dfrac{(x+3)^2}{16} + \dfrac{(y-1)^2}{9} = 1$
Center: $(-3, 1)$
Vertices: $(1, 1)$, $(-7, 1)$
Foci: $\left(-3 \pm \sqrt{7}, 1\right)$

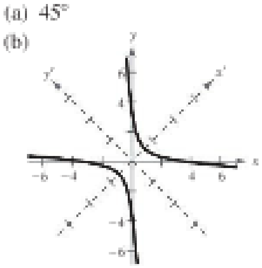


7. Circle: $(x - 2)^2 + (y - 1)^2 = \frac{1}{2}$
Center: $(2, 1)$

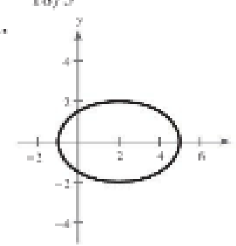


8. $(x - 2)^2 = \dfrac{4}{3}(y + 3)$ **9.** $\dfrac{y^2}{2/5} - \dfrac{x^2}{18/5} = 1$

10. (a) 45°
(b)



11.



$\dfrac{(x-2)^2}{9} + \dfrac{y^2}{4} = 1$

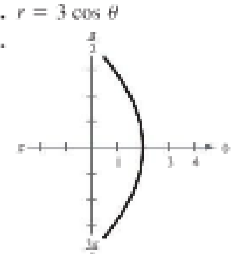
12. (a) $x = t$, $y = 3 - t^2$
(b) $x = t - 2$, $y = -t^2 + 4t - 1$

13. $\left(\sqrt{3}, -1\right)$

14. $\left(2\sqrt{2}, \dfrac{7\pi}{4}\right)$, $\left(-2\sqrt{2}, \dfrac{3\pi}{4}\right)$, $\left(2\sqrt{2}, -\dfrac{\pi}{4}\right)$

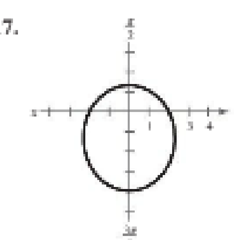
15. $r = 3\cos\theta$

16.



Parabola

17.



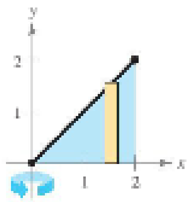
Ellipse

7.3 Exercises

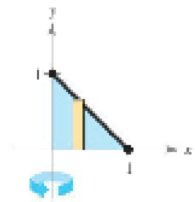
See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Finding the Volume of a Solid In Exercises 1–14, use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the y-axis.

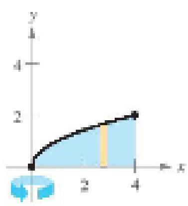
1. $y = x$



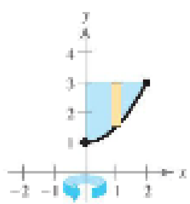
2. $y = 1 - x$



3. $y = \sqrt{x}$



4. $y = \frac{1}{2}x^2 + 1$



5. $y = \frac{1}{4}x^2, \quad y = 0, \quad x = 4$

6. $y = \frac{1}{2}x^3, \quad y = 0, \quad x = 3$

7. $y = x^2, \quad y = 4x - x^2$

8. $y = 9 - x^2, \quad y = 0$

9. $y = 4x - x^2, \quad x = 0, \quad y = 4$

10. $y = x^{3/2}, \quad y = 8, \quad x = 0$

11. $y = \sqrt{x - 2}, \quad y = 0, \quad x = 4$

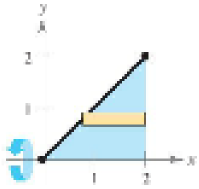
12. $y = -x^2 + 1, \quad y = 0$

13. $y = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \quad y = 0, \quad x = 0, \quad x = 1$

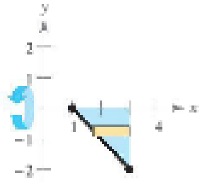
14. $y = \begin{cases} \frac{\sin x}{x}, & x > 0 \\ 1, & x = 0 \end{cases}, \quad y = 0, \quad x = 0, \quad x = \pi$

Finding the Volume of a Solid In Exercises 15–22, use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the x-axis.

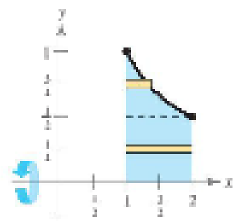
15. $y = x$



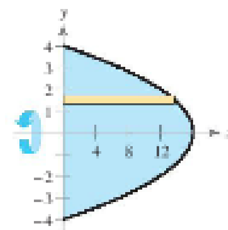
16. $y = 1 - x$



17. $y = \frac{1}{x}$



18. $x + y^2 = 16$



19. $y = x^3, \quad x = 0, \quad y = 8$

20. $y = 4x^2, \quad x = 0, \quad y = 4$

21. $x + y = 4, \quad y = x, \quad y = 0$

22. $y = \sqrt{x + 2}, \quad y = x, \quad y = 0$

Finding the Volume of a Solid In Exercises 23–26, use the shell method to find the volume of the solid generated by revolving the plane region about the given line.

23. $y = 2x - x^2, \quad y = 0, \quad$ about the line $x = 4$

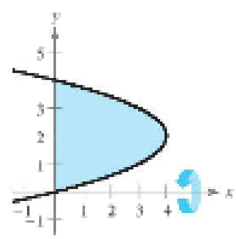
24. $y = \sqrt{x}, \quad y = 0, \quad x = 4, \quad$ about the line $x = 6$

25. $y = x^2, \quad y = 4x - x^2, \quad$ about the line $x = 4$

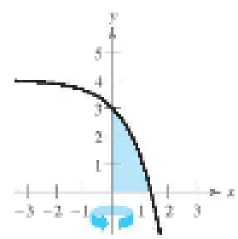
26. $y = \frac{1}{3}x^3, \quad y = 6x - x^2, \quad$ about the line $x = 3$

Choosing a Method In Exercises 27 and 28, decide whether it is more convenient to use the disk method or the shell method to find the volume of the solid of revolution. Explain your reasoning. (Do not find the volume.)

27. $(y - 2)^2 = 4 - x$



28. $y = 4 - e^x$



Choosing a Method In Exercises 29–32, use the disk method *or* the shell method to find the volumes of the solids generated by revolving the region bounded by the graphs of the equations about the given lines.

29. $y = x^3, \quad y = 0, \quad x = 2$

(a) the x-axis (b) the y-axis (c) the line $x = 4$

30. $y = \frac{10}{x^2}, \quad y = 0, \quad x = 1, \quad x = 5$

(a) the x-axis (b) the y-axis (c) the line $y = 10$

31. $x^{1/2} + y^{1/2} = a^{1/2}$, $x = 0$, $y = 0$

(a) the x-axis (b) the y-axis (c) the line $x = a$

32. $x^{2/3} + y^{2/3} = a^{2/3}$, $a > 0$ (hypocycloid)

(a) the x-axis (b) the y-axis

Finding the Volume of a Solid **In Exercises 33–36, (a) use a graphing utility to graph the plane region bounded by the graphs of the equations, and (b) use the integration capabilities of the graphing utility to approximate the volume of the solid generated by revolving the region about the y-axis.**

33. $x^{4/3} + y^{4/3} = 1$, $x = 0$, $y = 0$, first quadrant

34. $y = \sqrt{1 - x^3}$, $y = 0$, $x = 0$

35. $y = \sqrt[3]{(x-2)^2(x-6)^2}$, $y = 0$, $x = 2$, $x = 6$

36. $y = \dfrac{2}{1 + e^{1/x}}$, $y = 0$, $x = 1$, $x = 3$

WRITING ABOUT CONCEPTS

37. Representative Rectangles Consider a solid that is generated by revolving a plane region about the y-axis. Describe the position of a representative rectangle when using (a) the shell method and (b) the disk method to find the volume of the solid.

38. Describing Cylindrical Shells Consider the plane region bounded by the graphs of

$$y = k, \quad y = 0, \quad x = 0, \quad \text{and} \quad x = b$$

where $k > 0$ and $b > 0$. What are the heights and radii of the cylinders generated when this region is revolved about (a) the x-axis and (b) the y-axis?

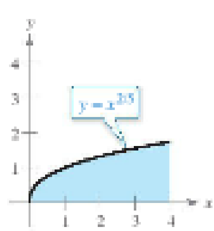
Comparing Integrals **In Exercises 39 and 40, give a geometric argument that explains why the integrals have equal values.**

39. $\pi \displaystyle\int_1^5 (x - 1)\,dx = 2\pi \int_0^2 y[5 - (y^2 + 1)]\,dy$

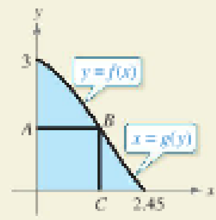
40. $\pi \displaystyle\int_0^2 [16 - (2y)^2]\,dy = 2\pi \int_0^4 x\left(\frac{x}{2}\right)dx$

41. Comparing Volumes The region in the figure is revolved about the indicated axes and line. Order the volumes of the resulting solids from least to greatest. Explain your reasoning.

(a) x-axis (b) y-axis (c) $x = 4$



42. HOW DO YOU SEE IT? Use the graph to answer the following.



(a) Describe the figure generated by revolving segment AB about the y-axis.

(b) Describe the figure generated by revolving segment BC about the y-axis.

(c) Assume the curve in the figure can be described as $y = f(x)$ or $x = g(y)$. A solid is generated by revolving the region bounded by the curve, $y = 0$, and $x = 0$ about the y-axis. Set up integrals to find the volume of this solid using the disk method and the shell method. (Do not integrate.)

Analyzing an Integral **In Exercises 43–46, the integral represents the volume of a solid of revolution. Identify (a) the plane region that is revolved and (b) the axis of revolution.**

43. $2\pi \displaystyle\int_0^2 x^3\,dx$

44. $2\pi \displaystyle\int_0^1 y - y^{3/2}\,dy$

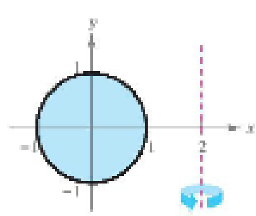
45. $2\pi \displaystyle\int_0^6 (y + 2)\sqrt{6 - y}\,dy$

46. $2\pi \displaystyle\int_0^1 (4 - x)e^x\,dx$

47. Machine Part A solid is generated by revolving the region bounded by $y = \frac{1}{2}x^2$ and $y = 2$ about the y-axis. A hole, centered along the axis of revolution, is drilled through this solid so that one-fourth of the volume is removed. Find the diameter of the hole.

48. Machine Part A solid is generated by revolving the region bounded by $y = \sqrt{9 - x^2}$ and $y = 0$ about the y-axis. A hole, centered along the axis of revolution, is drilled through this solid so that one-third of the volume is removed. Find the diameter of the hole.

49. Volume of a Torus A torus is formed by revolving the region bounded by the circle $x^2 + y^2 = 1$ about the line $x = 2$ (see figure). Find the volume of this "doughnut-shaped" solid. (*Hint:* The integral $\int_{-1}^{1} \sqrt{1 - x^2}\,dx$ represents the area of a semicircle.)



Section 7.3 *(page 462)*

1. $2\pi\int_0^2 x^2\,dx = \frac{16\pi}{3}$ **3.** $2\pi\int_0^4 x\sqrt{x}\,dx = \frac{128\pi}{5}$

5. $2\pi\int_0^4 \frac{1}{4}x^3\,dx = 32\pi$ **7.** $2\pi\int_0^2 x(4x-2x^2)\,dx = \frac{16\pi}{3}$

9. $2\pi\int_0^2 x(x^2-4x+4)\,dx = \frac{8\pi}{3}$

11. $2\pi\int_2^4 x\sqrt{x-2}\,dx = \frac{128\pi}{15}\sqrt{2}$

13. $2\pi\int_0^1 x\left(\frac{1}{\sqrt{2\pi}}e^{-x^2/2}\right)dx = \sqrt{2\pi}\left(1-\frac{1}{\sqrt{e}}\right) \approx 0.986$

15. $2\pi\int_0^2 y(2-y)\,dy = \frac{8\pi}{3}$

17. $2\pi\left[\int_0^{1/2} y\,dy + \int_{1/2}^1 y\left(\frac{1}{y}-1\right)dy\right] = \frac{\pi}{2}$

19. $2\pi\int_0^8 y^{4/3}\,dy = \frac{768\pi}{7}$

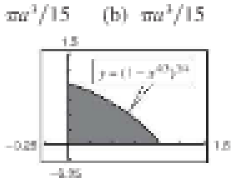
21. $2\pi\int_0^2 y(4-2y)\,dy = 16\pi/3$ **23.** 8π **25.** 16π

27. Shell method; it is much easier to put x in terms of y rather than vice versa.

29. (a) $128\pi/7$ (b) $64\pi/5$ (c) $96\pi/5$

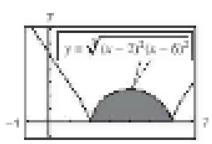
31. (a) $\pi a^3/15$ (b) $\pi a^3/15$ (c) $4\pi a^3/15$

33. (a)



(b) 1.506

35. (a)



(b) 187.25

37. (a) The rectangles would be vertical.
(b) The rectangles would be horizontal.

39. Both integrals yield the volume of the solid generated by revolving the region bounded by the graphs of $y = \sqrt{x-1}$, $y = 0$, and $x = 5$ about the x-axis.

41. a, c, b

43. (a) Region bounded by $y = x^2$, $y = 0$, $x = 0$, $x = 2$
(b) Revolved about the y-axis

45. (a) Region bounded by $x = \sqrt{6-y}$, $y = 0$, $x = 0$
(b) Revolved about $y = -2$

47. Diameter $= 2\sqrt{4-2\sqrt{3}} \approx 1.464$ **49.** $4\pi^2$

51. (a) Proof (b) (i) $V = 2\pi$ (ii) $V = 6\pi^2$ **53.** Proof

55. (a) $R_1(n) = n/(n+1)$ (b) $\lim_{n\to\infty} R_1(n) = 1$
(c) $V = \pi ab^{n+2}[n/(n+2)]$; $R_2(n) = n/(n+2)$
(d) $\lim_{n\to\infty} R_2(n) = 1$
(e) As $n \to \infty$, the graph approaches the line $x = b$.

57. (a) and (b) About 121,475 ft^3 **59.** $c = 2$

61. (a) $64\pi/3$ (b) $2048\pi/35$ (c) $8192\pi/105$

Section 7.4 *(page 473)*

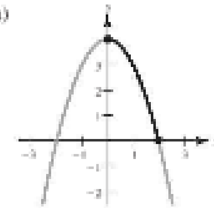
1. (a) and (b) 17 **3.** $\frac{5}{3}$ **5.** $\frac{2}{3}(2\sqrt{2}-1) \approx 1.219$

7. $5\sqrt{5} - 2\sqrt{2} \approx 8.352$ **9.** 309.3195

11. $\ln\left[(\sqrt{2}+1)/(\sqrt{2}-1)\right] \approx 1.763$

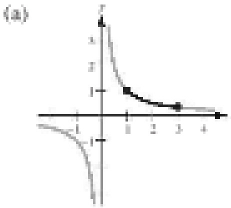
13. $\frac{1}{2}(e^2 - 1/e^2) \approx 3.627$ **15.** $\frac{76}{3}$

17. (a)



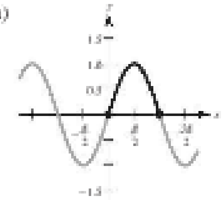
(b) $\int_0^2 \sqrt{1+4x^2}\,dx$
(c) About 4.647

19. (a)



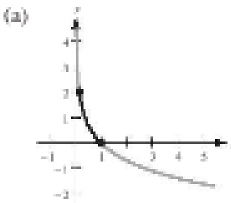
(b) $\int_1^3 \sqrt{1+\frac{1}{x^4}}\,dx$
(c) About 2.147

21. (a)



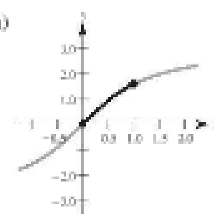
(b) $\int_0^{\pi} \sqrt{1+\cos^2 x}\,dx$
(c) About 3.820

23. (a)



(b) $\int_0^2 \sqrt{1+e^{-2y}}\,dy = \int_{e^{-2}}^1 \sqrt{1+\frac{1}{x^2}}\,dx$
(c) About 2.221

25. (a)



(b) $\int_0^1 \sqrt{1+\left(\frac{2}{1+x^2}\right)^2}\,dx$
(c) About 1.871

27. b

29. (a) 64.125 (b) 64.525 (c) 64.666 (d) 64.672

31. $20[\sinh 1 - \sinh(-1)] \approx 47.0$ m **33.** About 1480

35. $3\arcsin\frac{2}{3} \approx 2.1892$

37. $2\pi\int_0^3 \frac{1}{3}x^3\sqrt{1+x^4}\,dx = \frac{\pi}{9}(82\sqrt{82}-1) \approx 258.85$

39. $2\pi\int_1^2 \left(\frac{x^3}{6}+\frac{1}{2x}\right)\left(\frac{x^2}{2}+\frac{1}{2x^2}\right)dx = \frac{47\pi}{16} \approx 9.23$

41. $2\pi\int_{-1}^1 2\,dx = 8\pi \approx 25.13$

43. $2\pi\int_1^8 x\sqrt{1+\frac{1}{9x^{4/3}}}\,dx = \frac{\pi}{27}(145\sqrt{145}-10\sqrt{10}) \approx 199.48$

45. $2\pi\int_0^2 x\sqrt{1+\frac{x^2}{4}}\,dx = \frac{\pi}{3}(16\sqrt{2}-8) \approx 15.318$

47. 14.424

49. A rectifiable curve is a curve with a finite arc length.