Instructions for AP Calculus AB (Killheffer) Summer Work

1. The Summer Work is split into two sections – Trigonometry and Algebra

2. The Trigonometry section is due and should be dropped off at the school by July 25 in the front office at Cab.

3. The Algebra Section is due at our first class when we return from the summer

4. I will answer questions on the summer work for the first two days of class and then we will have a test on the summer work as the first summative for the marking period.

5. It is important that you understand all the concepts of the summer work as the course is built upon that understanding.

5 Math III Trig Supplement 2017 Name _____

Rationale for Needing a Supplement: (some parts adapted from Mrs. Cote)

The prerequisites listed by the College Board in the "Advanced Placement Course Description for Calculus AB" state: "...all students should ... study algebra, geometry, trigonometry, analytical geometry, and elementary functions. These functions include those that are linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise defined. In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on) and know the values of the trigonometric functions of common angles such as: 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$."

Review from Math II: In particular,

$$\sin x = \frac{\text{opposite leg}}{\text{hypotenuse}}, \qquad \cos x = \frac{\text{adjacent leg}}{\text{hypotenuse}}, \qquad \tan x = \frac{\text{opposite leg}}{\text{adjacent leg}}.$$

You should also know the following three functions:

Cosecant is $\csc x = \dfrac{1}{\sin x}$ so it can be written as $\csc x = \dfrac{\text{hypotenuse}}{\text{opposite leg}}$.

Secant is $\sec x = \dfrac{1}{\cos x}$ so it can be written as $\csc x = \dfrac{\text{hypotenuse}}{\text{adjacent leg}}$.

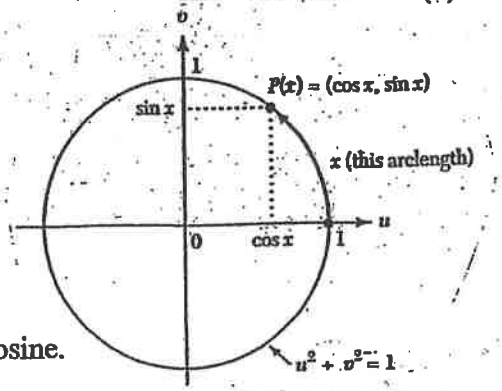
Cotangent is $\cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x} = \dfrac{\text{adjacent leg}}{\text{opposite leg}}$.

Now you need to view sine, cosine, tangent, as well as, cosecant, secant, and cotangent as functions in their own right.

Definition of sine and cosine: For any real number, x, let P(x) be the point reached by moving x units of distance counterclockwise around the unit circle, starting from (1, 0). (If x < 0, go clockwise.) Then

cos x = first coordinate of P(x) sin x = second coordinate of P(x)

DO ALL



1) So now let's find some values for sine and cosine.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{-\pi}{2}$
P(x)						
Sin x						
Cos x						

2) Now get the multiples of $\frac{\pi}{4}$ from the unit circle.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$
P(x)								
Sin x								
Cos x								

3) Now get the multiples of $\frac{\pi}{6}$ from the unit circle.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$
P(x)								
Sin x								
Cos x								

DO ALL

Keep going until you get to 2π.

x								
P(x)								
Sin x								
Cos x								

Now we'll learn a short cut so you can memorize the values you need to know.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sin x					
Cos x					
Tan x					

Assignment 1

Without the use of any calculator (or unit circle), fill in the exact values for each of the following. Simplify all answers.

1) $\cos \frac{5\pi}{6}$

2) $\sin \frac{5\pi}{6}$

3) $\sec \frac{5\pi}{6}$

4) $\csc \frac{5\pi}{6}$

5) $\tan \frac{5\pi}{6}$

6) $\cot \frac{5\pi}{6}$

7) $\sin \frac{5\pi}{3}$

8) $\sec \frac{5\pi}{3}$

9) $\csc \frac{5\pi}{3}$

10) $\tan \frac{\pi}{2}$

11) $\tan \frac{3\pi}{2}$

12) $\sin \frac{\pi}{2}$

13) $\cos \frac{\pi}{2}$

14) $\cos \frac{3\pi}{2}$

15) $\cos \frac{-\pi}{2}$

16) $\cos \frac{\pi}{4}$

17) $\cos \frac{-3\pi}{4}$

18) $\tan \frac{3\pi}{4}$

DO ALL

19) $\tan \frac{5\pi}{4}$ 20) $\tan \frac{7\pi}{4}$ 21) $\cos \frac{2\pi}{3}$

22) $\sin \frac{2\pi}{3}$ 23) $\sin \frac{4\pi}{3}$ 24) $\tan \pi$

25) $\sec \frac{\pi}{2}$ 26) $\csc \frac{\pi}{2}$

*Memorize the definitions of sine and cosine.

*Memorize the quick chart for the values of sine, cosine, and tangent of common angles: $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$.

II. Fundamental Properties and Identities of Trigonometric Functions

1) Even and Odd functions

Definition:

An **even** function, f(x), is a function such that for every x in its domain, f(-x) = f(x). (You can rotate it about the y axis and get the same graph.)

An **odd** function, f(x), is a function such that for every x in its domain, f(-x) = -f(x). (You can rotate it through the origin and get the same graph.)

Determine if the six trig functions are even or odd or neither.

2) Periodic –

Determine the period of sine and cosine.

ODDS ONLY

Trig 2 Name: _____

Table/Seat: ____ Date: _____

1. Find the exact value of the following functions.

a) $\sin\frac{5\pi}{6}$ b) $\tan\frac{5\pi}{3}$ c) $\cos\frac{\pi}{4}$

d) $\sin\pi$ e) $\tan\frac{7\pi}{6}$ f) $\cos\frac{2\pi}{3}$

g) $\cos\frac{5\pi}{6}$ h) $\tan\frac{3\pi}{4}$ i) $\sin\frac{\pi}{4}$

j) $\cos\pi$ k) $\tan\frac{4\pi}{3}$ l) $\sin\frac{7\pi}{6}$

2. Write the letter of the correct identity next to each expression. Draw a diagram to justify your choice.

_____	$\cos(-x)$	_____	$\sin\left(\frac{\pi}{2}+x\right)$	a. $\sin x$
_____	$\sin(-x)$	_____	$\cos(\pi-x)$	b. $-\sin x$
_____	$\sin^2 x+\cos^2 x$			c. $\cos x$
_____	$\cos\left(\frac{\pi}{2}-x\right)$	_____	$\sin(\pi-x)$	d. $-\cos x$
_____	$\sin\left(\frac{\pi}{2}-x\right)$	_____	$\sin(\pi+x)$	e. 1
_____	$\cos\left(\frac{\pi}{2}+x\right)$	_____	$\cos\left(\frac{3\pi}{2}-x\right)$	

Trig 3 Name: _____ ODDS

Table/Seat: ____

NON-Calculator

I. Write each of the following circular functions in terms of $\cos(x)$ and $\sin(x)$.

1. $\tan(x)$ 2. $\cot(x)$

3. $\csc(x)$ 4. $\sec(x)$

II. Find the exact value for each of the following problems.

5. If $\sin(x) = -\frac{3}{5}$ and x is in quadrant 3, what is $\tan(x)$? $\sec(x)$?

6. If $\tan(x) = \frac{3}{4}$ and x is in quadrant 3, what is $\cot(x)$? $\csc(x)$? $\sin(x)$?

7. If $\sec(x) = \frac{5}{3}$ and x is in quadrant 4, what is $\cot(x)$? $\csc(x)$? $\sin(x)$?

III. Write the exact value for each of the following.

8. $\csc\left(\frac{\pi}{3}\right)$ 9. $\tan\left(\frac{\pi}{2}\right)$

10. $\sec\left(\frac{2\pi}{3}\right)$ 11. $\cot\left(\frac{\pi}{2}\right)$

12. $\cot\left(\frac{3\pi}{4}\right)$ 13. $\csc\left(\frac{\pi}{2}\right)$

14. $\sec\left(\frac{5\pi}{4}\right)$ 15. $\sec\left(\frac{5\pi}{6}\right)$

16. $\cot\left(\frac{4\pi}{3}\right)$ 17. $\sec\left(\frac{11\pi}{6}\right)$

ALL

V. Inverse Functions: Their Meaning, Domains, Ranges

1. a) Try solving $2 \sin x - 1 = 0$ without a calculator. You should get two values between 0 and 2π. Now try it on the calculator. Notice there is only one value. Why?

b) Graph of $y = \sin^{-1} x$:

c) Relating the graph above to the answer of $2 \sin x - 1 = 0$.

d) Writing the answers to $2 \sin x - 1 = 0$.

2. a) Try solving $2 \cos^2 x = \cos x$ where x is between 0 and 2π.

b) Look at the graph of $y = \cos^{-1} x$. What do you think the calculator answer will be? Try it!

ALL

VII. Solving Trig Equations

Examples:

1) $2 \sin x - 1 = 0$

2) **Collecting Like Terms**
$\sin x + \sqrt{2} = -\sin x$

3) **Extracting Square Roots**

$3 \tan^2 x - 1 = 0$

4) **Factoring.** $\cot x \cos^2 x = 2 \cot x$

5) **Factoring** Find all solutions of

$2 \sin^2 x - \sin x - 1 = 0$ in the interval $0 \leq x \leq 2\pi$

6) **Writing in Terms of Single Trig Function**
$2\sin^2 x + 3\cos x - 3 = 0$

7) **Squaring and Converting to Quadratic Type**
Find all solutions of $\cos x + 1 = \sin x$ in the interval $0 \leq x \leq 2\pi$

DO ODDS ONLY

8) Using Inverse Functions
Find all solutions of $\sec^2 x - 2\tan x = 4$.

9) Using the Quadratic Formula
Find all solutions of $\sin^2 t - 3\sin t - 2 = 0$ in the interval $0 \le t \le 2\pi$

Exercises Section VII

Find all exact solutions of the equation in the interval $0 \le x \le 2\pi$.

1. $2\cos x + 1 = 0$

~~2. $2\sin x - 1 = 0$~~

3. $\sqrt{3}\csc x - 2 = 0$

~~4. $\tan x + 1 = 0$~~

5. $2\sin^2 x = 1$

~~6. $\tan^2 x = 3$~~

7. $3\sec^2 x - 4 = 0$

~~8. $\csc^2 x - 2 = 0$~~

9. $\tan x(\tan x - 1) = 0$

~~10. $\cos x(2\cos x + 1) = 0$~~

11. $\sin x(\sin x + 1) = 0$

~~12. $4\sin^2 x - 3 = 0$~~

13. $\sin^2 x = 3\cos^2 x$

~~14. $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$~~

Find all solutions of the equation in the interval $0 \le x \le 2\pi$.

15. $\sec x \csc x - 2\csc x = 0$

~~16. $\sec^2 x - \sec x - 2 = 0$~~

17. $2\sin^2 x + 3\sin x + 1 = 0$

~~18. $3\tan^3 x - \tan x = 0$~~

19. $\cos^3 x = \cos x$

~~20. $4\sin^3 x + 2\sin^2 x - 2\sin x - 1 = 0$~~

21. $2\sec^2 x + \tan^2 x - 3 = 0$

~~22. $2\sin^2 x = 2 + \cos x$~~

23. $2\sin x + \csc x = 0$

~~24. $\csc x + \cot x = 1$~~

25. $2\tan^2 x + 7\tan x - 15 = 0$

~~26. $12\cos^2 x + 5\cos x - 3 = 0$~~

27. $12\sin^2 x - 13\sin x + 3 = 0$

~~28. $3\tan^2 x + 4\tan x - 4 = 0$~~

29. $6\cos^2 x - 13\cos x + 6 = 0$

~~30. $4\cos^2 x - 4\cos x - 1 = 0$~~

QUIZ 4 HERE!

Answers for page 15

1. $\frac{2\pi}{3}, \frac{4\pi}{3}$

2. $\frac{\pi}{6}, \frac{5\pi}{6}$

3. $\frac{\pi}{3}, \frac{2\pi}{3}$

4. $\frac{3\pi}{4}, \frac{7\pi}{4}$

5. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

6. $\frac{\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$

7. $\frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

8. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

9. $0, \pi, 2\pi, \frac{\pi}{4}, \frac{5\pi}{4}$

10. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$

11. $0, \pi, 2\pi, \frac{3\pi}{2}$

12. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

13. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

14. $\frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$

15. $\frac{\pi}{3}, \frac{5\pi}{3}$

16. $\frac{\pi}{3}, \frac{5\pi}{3}, \pi$

17. $\frac{11\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}$

18. $0, \pi, 2\pi, \frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

19. $0, \pi, 2\pi, \frac{\pi}{2}, \frac{3\pi}{2}$

20. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{7\pi}{6}, \frac{11\pi}{6}$

21. $\frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

22. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$

23. No Solutions

24. $\frac{\pi}{2}, \frac{3\pi}{2}$

25. 0.98, 4.12, 4.9, 1.77

26. 1.23, 5.05, 2.41, 3.86

27. 0.34, 2.80, 0.85, 2.29

28. 0.59, 3.73, 5.17, 2.03

29. 0.84, 5.44

30. 1.78, 4.50

ALL

VIII. Trig Identities

Examples: Verify that each equation is an identity. Remember: Work only one side of the equation!

1) $\cos\theta = \dfrac{\cot\theta}{\csc\theta}$

2) $\dfrac{\tan^2 A + 1}{\sec A} = \sec A$

3) $\cot x + \tan x = \sec x \csc x$

4) $\dfrac{\cos A}{\sec A} + \dfrac{\sin A}{\csc A} = \sec^2 A - \tan^2 A$

5) $\sin^2 A \sec^2 A + \sin^2 A \csc^2 A = \sec^2 A$

6) $\tan^2\theta \sin^2\theta = \tan^2\theta + \cos^2\theta - 1$

ALL

7) $\dfrac{\sin\theta}{1-\cos\theta}-\dfrac{\sin\theta\cos\theta}{1+\cos\theta}=\csc\theta(1+\cos^2\theta)$

8) $(\sec A+\csc A)(\cos A-\sin A)=\cot A-\tan A$

STEPS IN PROVING IDENTITIES

1. Pick the member you wish to work with and write it down. Usually it is easier to start with the more complicated side.
2. Look for *algebraic* thing to do.
   a. If there are two terms and you want only one,
      i. add fractions,
      ii. factor something out.
   b. Multiply by a clever form of 1
      i. multiply a numerator or denominator by its conjugate,
      ii. to get a desired expression in numerator or denominator.
   c. Do any obvious algebra such as distributing, squaring, or multiplying polynomials.
3. Look for *trig* things to do.
   a. Look for familiar trig expressions. (Think all identities!)
   b. If there are squares of functions, think of Pythagorean Identities.
   c. Reduce the number of trig functions, maybe changing everything to sin and cos.
4. Keep looking at the desired result to make sure you are headed in the

Exercises:

Verify that each equation is an identity.
Remember: Work only one side of the equation!

1. $\tan A=\dfrac{\sec A}{\csc A}$

2. $\sec x-\tan x=\dfrac{1-\sin x}{\cos x}$

3. $\sec x\csc x=\tan x+\cot x$

4. $(\sin A+\cos A)^2=\dfrac{2+\sec A\csc A}{\sec A\csc A}$

ODDS ONLY

5. $\frac{\cos y}{1-\sin y}=\frac{1+\sin y}{\cos y}$

6. $\csc x-1=\frac{\cot^2 x}{\csc x+1}$

7. $\sin A\cos A\tan A+\cos^2 A=1$

8. $\sin x+\cos x=\frac{\cos x}{1-\tan x}+\frac{\sin x}{1-\cot x}$

9. $\cos A=\sin A\cot A$

10. $\frac{1+\tan x}{\sin x+\cos x}=\sec x$

11. $\sin A+\cos A=\frac{2\sin^2 A-1}{\sin A-\cos A}$

12. $(\sin A-1)(\tan A+\sec A)=-\cos A$

13. $\cos A\cos(-A)-\sin A\sin(-A)=1$

14. $\cos B\cot B=\csc B-\sin B$

15. $(\csc x-\cot x)^2=\frac{1-\cos x}{1+\cos x}$

16. $\sin A+\cos A+\tan A\sin A=$
$\sec A+\cos A\tan A$

17. $\cos\left(\frac{\pi}{2}+x\right)=-\sin x$

18. $\cos(60^\circ+A)=\sin(30^\circ-A)$

19. $\sin(A+\pi)=-\sin A$

20. $\cos(\pi+x)=-\cos x$

21. $\tan(x+45^\circ)=\frac{1+\tan x}{1-\tan x}$

22. $\sin(A+B)=\frac{\tan A+\tan B}{\sec A\sec B}$

23. $\cos(A+B)=\frac{1-\tan A\tan B}{\sec A\sec B}$

24. $\sec(A-B)=\frac{\sec A\sec B}{1+\tan A\tan B}$

25. $\sin(x+y)\sin(x-y)=\sin^2 x-\sin^2 y$

26. $\csc 2A=\frac{1}{2}\sec A\csc A$

27. $\cos A-\sin A=\frac{\cos 2A}{\cos A+\sin A}$

28. $(\sin A+\cos A)^2-1=\sin 2A$

29. $\cos x-1=\frac{\cos 2x-1}{2(\cos x+1)}$

30. $\sec 2A=\frac{\cos^2 A+\sin^2 A}{\cos^2 A-\sin^2 A}$

31. $\tan\frac{A}{2}=\frac{\sin A}{1+\cos A}$

32. $\sin 3x=3\sin x-4\sin^3 x$

33. $\cos 3x=4\cos^3 x-3\cos x$

34. $\tan^2 x=\frac{\sin^2 x}{\cos^4 x+\cos^2 x\sin^2 x}$

35. $\sin^4 A+\cos^2 A=\cos^4 A+\sin^2 A$

QUIZ 5 & TEST!

5 Math III Name ODDS ONLY Color

More Verifying Identities

NO CALCULATORS!

Verify that each equation is an identity. Remember: Work only one side of the equation. Show All Work!

1. $\frac{\cot\theta}{\csc\theta} = \cos\theta$

2. $\frac{\tan\theta}{\sec\theta} = \sin\theta$

3. $\frac{1-\sin^2\theta}{\cos\theta} = \cos\theta$

4. $\frac{\tan^2\theta+1}{\sec\theta} = \sec\theta$

5. $\cot\theta + \tan\theta = \sec\theta\csc\theta$

6. $\frac{\cos\theta}{\sec\theta} + \frac{\sin\theta}{\csc\theta} = \sec^2\theta - \tan^2\theta$

7. $\sin^4\theta - \cos^4\theta = 2\sin^2\theta - 1$

8. $(1-\cos^2\theta)(1+\cos^2\theta) = 2\sin^2\theta - \sin^4\theta$

*9. $\frac{\cos x+1}{\tan^2 x} = \frac{\cos x}{\sec x - 1}$

10. $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$

*11. $\frac{\tan\theta}{1+\cos\theta} + \frac{\sin\theta}{1-\cos\theta} = \cot\theta + \sec\theta\csc\theta$

12. $\frac{\cot^2 x - 1}{1+\cot^2 x} = 1 - 2\sin^2 x$

13. $\sin^2\theta\sec^2\theta + \sin^2\theta\csc^2\theta = \sec^2\theta$

14. $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

**15. $\sin x + \cos x = \frac{\sin x}{1-\frac{\cos x}{\sin x}} + \frac{\cos x}{1-\frac{\sin x}{\cos x}}$

16. $\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta} = 4\cot\theta\csc\theta$

17. $(\sec A + \csc A)(\cos A - \sin A) = \cot A - \tan A$

18. $\sin^2 A + \tan^2 A + \cos^2 A = \sec^2 A$

19. $\frac{\sin^2 x}{\cos x} = \sec x - \cos x$

*20. $\frac{(\sec x - \tan x)^2 + 1}{\sec x\csc x - \tan x\csc x} = 2\tan x$

* - challenging

** - most challenging!

Summary of All Identities In One Place! (Yes you can thank me!)

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities

$\frac{\sin\theta}{\cos\theta} = \tan\theta$ $\frac{\cos\theta}{\sin\theta} = \cot\theta$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Cofunction Identities

$\sin\theta = \cos(90° - \theta)$ $\cos\theta = \sin(90° - \theta)$

$\tan\theta = \cot(90° - \theta)$ $\cot\theta = \tan(90° - \theta)$

$\sec\theta = \csc(90° - \theta)$ $\csc\theta = \sec(90° - \theta)$

Sum and Difference Identities

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Opposite-Angle Identities

$\sin(-A) = -\sin A$ $\cos(-A) = \cos A$

Double-Angle Identities

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

Half-Angle Identities

$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}, \cos\alpha \neq -1$

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Modeling Periodic Data

Since the graphs of the trig functions are repetitious (periodic), they are called *sinusoids*. The prefix "sinus-" is pronounced like what gets stopped up when you have a cold. The suffix "-oid" means "like." Certain features of these graphs are given special names, as shown in Figure 1a below.

A **cycle** of a periodic function is a portion of the graph from one point to the point at which the graph starts repeating itself.

The **period**, p, of a periodic function is the change in x corresponding to one cycle. That is, $f(x+p) = f(x)$ for all values of x.

The **amplitude** of a sinusoid is the distance from its axis to a high point or a low point.

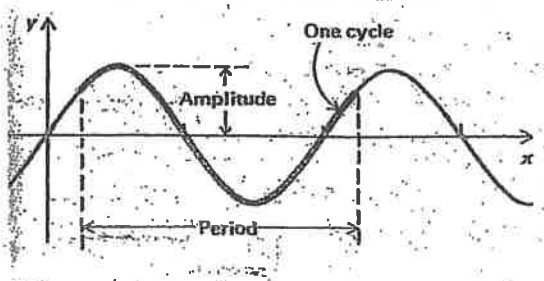


Figure 1a

Many periodic phenomena have graphs that look like sinusoids. For example, the time of sunrise as a function of the day of the year has a graph that looks like Figure 1b below. In order to use sinusoidal functions as mathematical models for these phenomena, you must be able to graph sinusoids that have periods other than 2π and amplitudes other than 1. You must also be able to position the graph away from the horizontal axis.

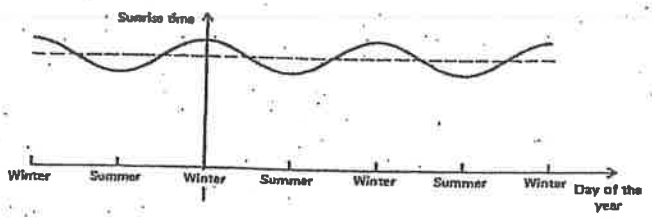


Figure 1b

Figure 1c below shows the graph of $y = 3\sin 2x$. The graph of $y = \sin x$ is also shown for comparison. The coefficient of 3 is the "vertical stretch" of the graph (or vertical shift). The factor of 2 in the argument of 2x tells the number of cycles the graphs makes in 2π units of x.

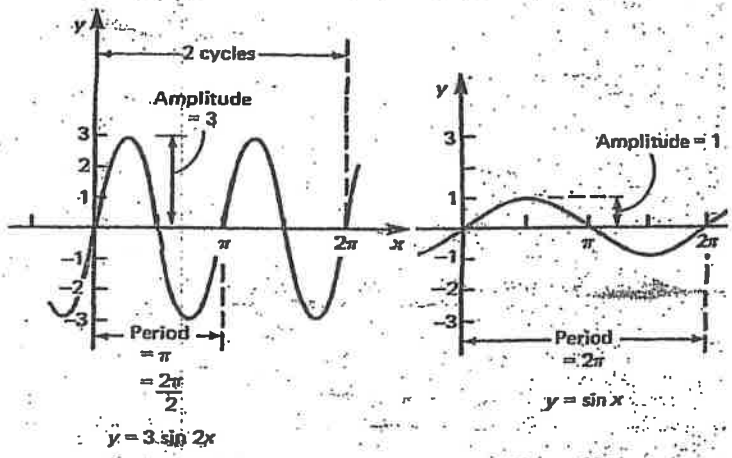


Figure 1c

Suppose you are to graph $y = \cos(x - \frac{\pi}{3})$. To find critical points on the graph, you need to find values of x that make the argument $(x - \frac{\pi}{3})$ equal to $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$, etc. (ie: Values that would be easily graphed.) Therefore,

$x - \frac{\pi}{3} = 0$

$x = \frac{\pi}{3}$

Since $\cos 0 = 1$, the graph is a *high* point when $x = \frac{\pi}{3}$.

$x - \frac{\pi}{3} = \frac{\pi}{2}$

$x = \frac{5\pi}{6}$

Since $\cos(\frac{\pi}{2}) = 0$, the graph crosses the x-axis at $x = \frac{5\pi}{6}$.

The graph is congruent to that of $y = \cos x$, but is horizontally *displaced* $\frac{\pi}{3}$ units to the right.

Conclusion: If $y = \cos(x - D)$, then the graph is the same as $y = \cos x$ but displaced D units horizontally. See figure 2a on the following page.

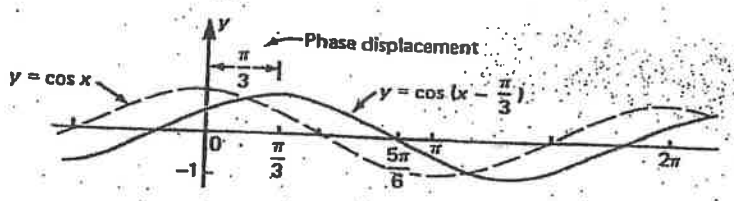


Figure 2a

Now let's look at y = 3 + cos x. Each point will be 3 units *higher* than the corresponding point on y = cos x. See figure 2b below. **Conclusion:** If y = C + cos x, then the *sinusoidal axis* is C units above the x-axis.

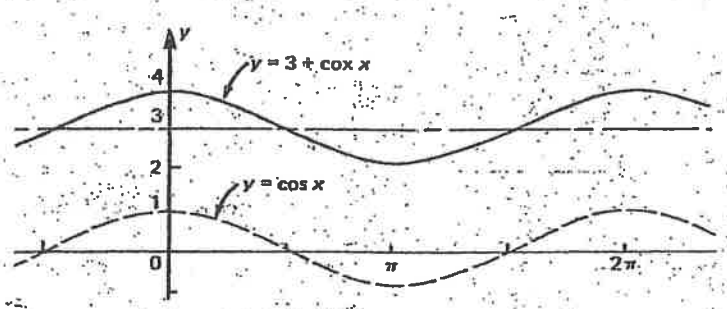


Figure 2b

Putting it all together! The general sinusoidal equation is

$$y = C + A \cos B(x - D)$$

A graph of this equation is shown in Figure 2c below.

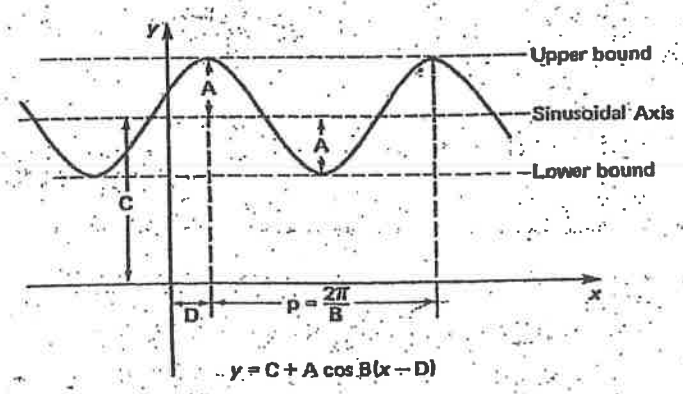


Figure 2c

1. |A| is the **amplitude** (absolute value is necessary because A may be a negative number, but amplitude is positive). If A is negative, the cosine starts a cycle at a low point, and the sine starts a cycle on the axis but going downward.

2. |B| is the number of cycles the sinusoid makes in 2π units of x, so the **period** is $p = 2\pi/|B|$.

3. C is the **vertical shift.** It may be positive or negative.

4. D is the **phase displacement (shift horizontal).** It may be positive or negative.

A cycle "starts" when the argument is 0, or a multiple of 2π. Since cos 0 = 1 and sin 0 = 0, the cosine graph starts a cycle at a *high* point and the sine function starts a cycle at a *midpoint*, going up.

Example 1: Sketch the graph of
$y = 5 + 3 \cos \frac{1}{4}(x + \pi)$.

Step by step procedure:
1. Draw the sinusoidal axis at y = 5.
2. Since the amplitude is 3, draw upper and lower bounds by going 3 units above and below the sinusoidal axis.
3. Find the starting point of a cycle at $x = -\pi$, the horizontal shift. Cosine starts a cycle at a *high* point.
4. The period is $2\pi/(1/4)$ which is 8π. So the cycle will end 8π units down the x-axis, at $x = 7\pi$.
5. Halfway between these two high points there is a low point. Halfway between each high and low point the graph crosses the sinusoidal axis.

See Figure 2d below.

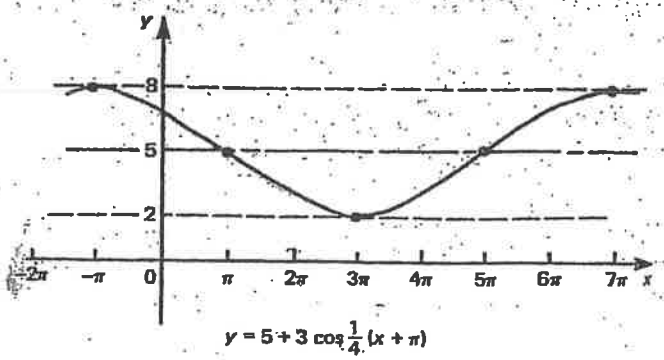


Figure 2d

Example 2: Sketch the graph of

$$y = -2 + 4 \sin \frac{\pi}{5}(x - 3).$$

The period is: $p = 2\pi \div (\pi/5) = 10$. The sinusoidal axis is at $y = -2$. The upper and lower bounds are 4 units above and below this sinusoidal axis. A cycle starts at $x = 3$, the horizontal shift. Since this is the *sine* function, the graph starts at a *midpoint*, going up. The end of this cycle is at $x = (3 + 10)$ or 13. Again, there are critical points each ¼ cycle. The graph is show below in Figure 3a.

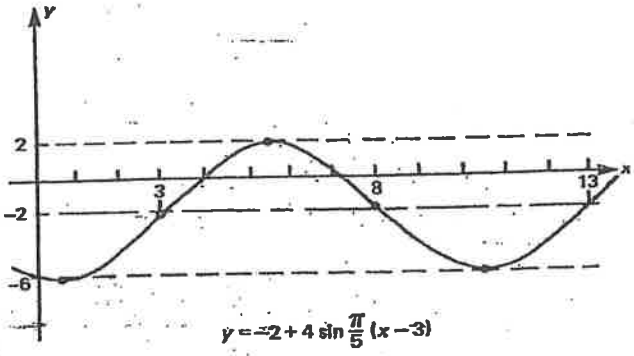


Figure 3a

Homework Set #1: Sketch the graph of each function.

1. $y = 3 + 2 \cos \frac{1}{5}(x - \pi)$

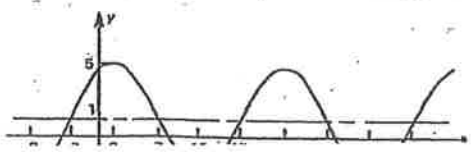
2. $y = -4 + 5 \sin \frac{2}{3}(x + \frac{\pi}{2})$

3. $y = -10 + 20 \cos \frac{\pi}{3}(x - 1)$

4. $y = 3 + 5 \sin \frac{\pi}{4}(x - 3)$

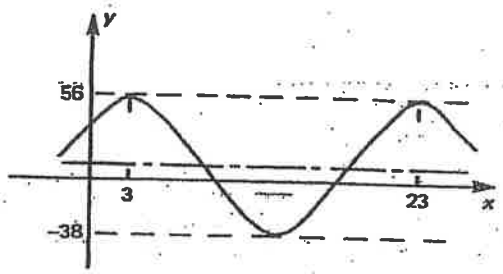
5. $y = 100 + 150 \cos \pi(x + 0.7)$

6. By reversing the graphing process you did above, you should be able to find the particular equation of a sinusoid from a given graph. Write the particular equation of the following sinusoid:



PART II: Now you are ready to use sinusoidal functions as mathematical models. In order to do this, you must be able to write the equation that has the right period, amplitude, horizontal and vertical shifts.

Example 3: Write the particular equation of the sinusoid below.



Your reasoning might follow as:

1. It is easier to use the *cosine* function since a cycle starts at a *high* point. So the general equation is

$$y = C + A \cos B(x - D).$$

2. One cycle starts at $x = 3$ and ends at $x = 23$. So the period is $p = 23 - 3$ or 20. Therefore, $B = 2\pi / p = 2\pi / 20 = \pi / 10$.

3. The sinusoidal axis is halfway between the upper bound, 56, and the lower bound, -38. So the vertical shift is the <u>average</u> of 56 and -38. Therefore, $C = ½ (56 + (-38)) = ½ (18)$ or 9.

4. The amplitude is the distance between the sinusoidal axis and the upper bound. Therefore, $A = 56 - 9$ or 47.

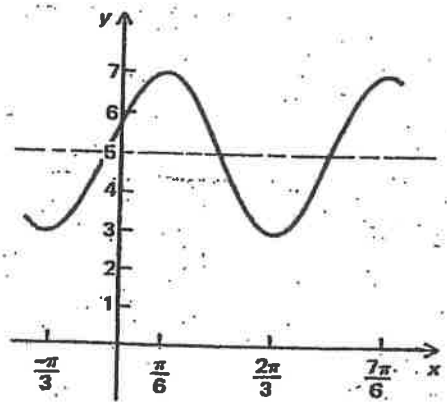
5. Using the cosine function, the horizontal shift is 3. Therefore, $D = 3$.

6. Therefore the equation is:

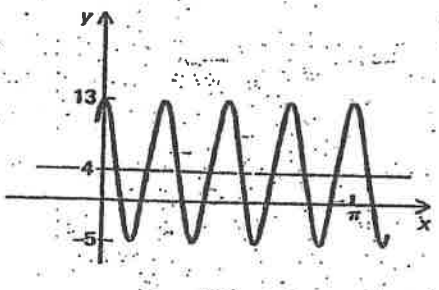
$$y = 9 + 47 \cos \frac{\pi}{10}(x - 3).$$

Homework Set #2: Write the equation of each sketched sinusoidal.

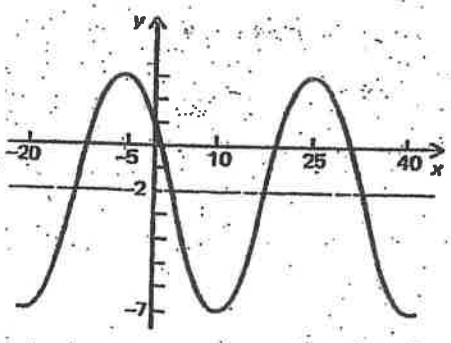
1.



2.



3.



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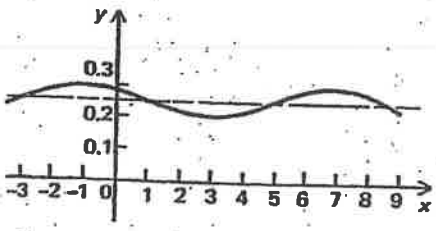

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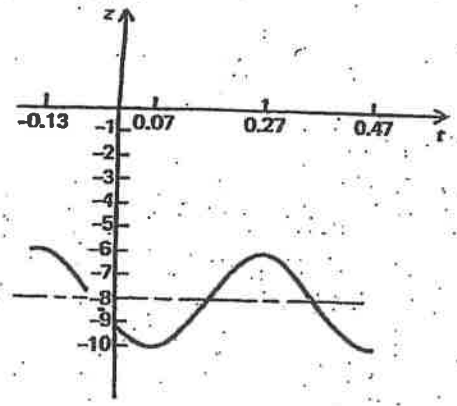
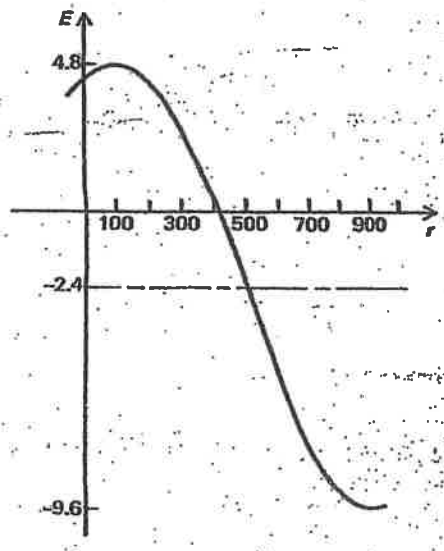


6.



For Problems 7 – 10, sketch the graph of the sinusoidal described (if necessary) and write the particular equation.

7. Period = 12, amplitude = 7, horizontal shift (for the cosine) = 5, vertical shift = 3.

8. Period = 0.01, amplitude = 8, horizontal shift (for the cosine) = 0.02, vertical shift = -5.

9. Low point at (x, y) = (2, -1), next high point at (5, 4).

10. High point at (r, s) = (-5, -3), next low point at (-1, -13).

PART III: You should be able to come up with several real-world situations in which a dependent variable repeated its value at regular intervals as the independent variable changed. For example,

1. The depth of the water at the beach depends on the time of day due to the motion of the tides.
2. The distance required to stop your car depends on how fast you were going when you applied the brakes.
3. The temperature of a cup of coffee depends on how long it has been since the coffee was poured.
4. A gymnast is jumping up and down on a trampoline. Her distance from the floor depends on the time.
5. The distance you go depends on how long you have been going (at a constant speed).
6. As you ride the Ferris wheel, your distance from the ground depends on how long you have been riding.
7. The average temperature for any particular day (averaged over many years) depends on the day of the year.
8. A pendulum swings back and forth in a grandfather clock. The distance from the end of the pendulum to the left side of the clock depends on time.
9. A straight line starts along the positive x-axis and rotates counterclockwise around and around the origin of a Cartesian coordinate system. The *slope* of the line depends on the number of degree through which the line has rotated.
10. The volume of air in your lungs varies periodically with time as you breathe. A reasonable sketch of the graph of this function is shown below in Figure 5a.

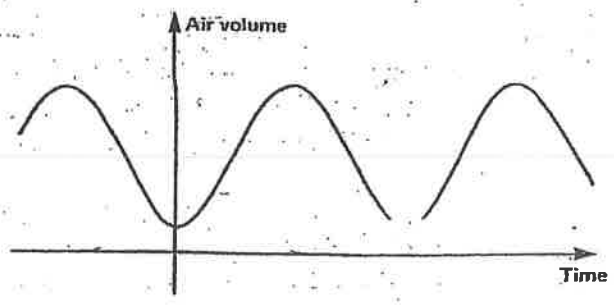


Figure 5a

Example 4: Suppose the waterwheel in Figure 5b below rotates at 6 revolutions per minute (rpm). You start your stopwatch. Two seconds later, point P on the rim of the wheel is at its greatest height. You are to model the distance d of point P from the surface of the water in terms of the number of seconds t the stopwatch reads.

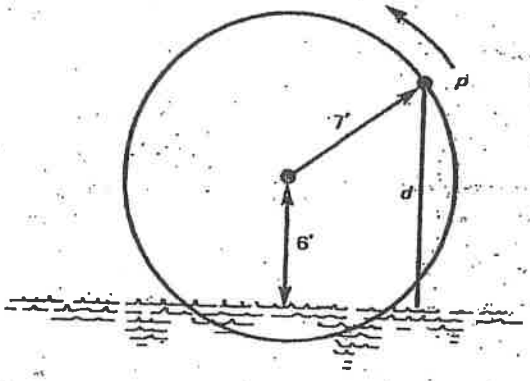


Figure 5b

Assuming that d varies sinusoidally with t, you can sketch a graph as in Figure 5c below.

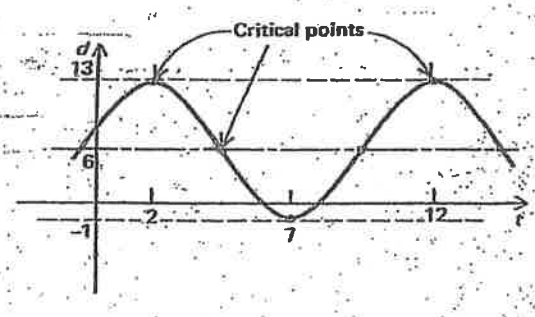


Figure 5c

Your thought process might be:

1. The sinusoidal axis is 6 units above the t-axis, because the center of the waterwheel is 6 feet above the surface of the water.

2. The amplitude is 7 units, since the point P goes 7 feet above and 7 feet below the center of the wheel.

3. Therefore, the upper and lower bounds of the graph are 6 + 7 or 13, and 6 – 7 or -1.

4. The point P was at its highest when the stopwatch reads 2 seconds. Thus the horizontal shift (for cosine) is 2 units.

5. The period is 10 seconds, since the waterwheel makes 6 complete revolutions every 60 seconds (1 minute).

6. Therefore, the sinusoid reaches its next high point at 2 + 10 or 12 units on the t-axis.

7. Halfway between two high points there is a low point at t = ½ (2 + 12) or 7; halfway between each high and low point the graph crosses the sinusoidal axis.

8. With the critical points from above, you can sketch the graph, if you'd like.

From the graph, the four constants in the sinusoidal equation are

$A = 7$, $B = 2\pi / p = 2\pi / 10 = \pi/5$, $C = 6$ and $D = 2$.

Therefore, the equation is

$$d = 6 + 7 \cos (\pi/5)(t-2).$$

The equation can be used to make predictions of d for given values of t. For example, to find out how far P is from the water when t = 5.5, you would simply substitute 5.5 for t in the equation to get d is approximately 1.8854. So the point is about 1.9 feet above the water when t = 5.5.

Homework Set #3:

1. **Ferris Wheel Problem.** As you ride the Ferris wheel, your distance from the ground varies sinusoidally with time. When the last seat is filled and the Ferris wheel starts, your seat is at the position shown in Figure 6a above. Let t be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 feet above the ground, and that the wheel makes a revolution once every 8 seconds. The diameter of the wheel is 40 feet.

a. Sketch a graph of this sinusoid.

b. What is the lowest you go as the Ferris wheel turns and why is this number greater than zero?
c. Write the particular equation of this sinusoid.
d. Predict your height above the ground when t = 6, t = 4 1/3, t = 9 and t = 0.

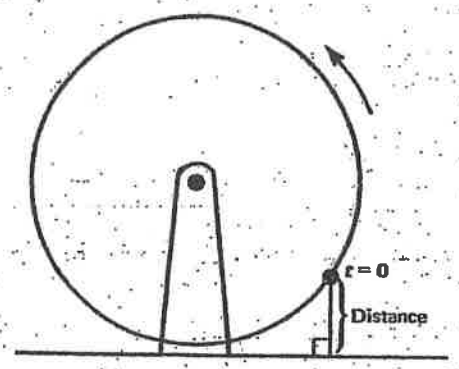


Figure 6a

2. **Tidal Wave Problem.** A tsunami (commonly called a "tidal wave" because its effect is like a rapid change in tide) is a fast-moving ocean wave caused by an underwater earthquake. The water first goes down from its normal level, then rises an equal distance above its normal level, and finally returns to its normal level. The period is about 15 minutes.

Suppose that a tsunami with an amplitude of 10 meters approaches the pier at Honolulu, where the normal depth of the water is 9 m.

a. Assuming that the depth of the water varies sinusoidally with time as the tsunami passes, predict the depth of the water at the following times after the tsunami first reaches the pier: t = 2 min, 4 min & 12 min.

b. According to your model, what will the *minimum* depth of the water be? How do you interpret this answer in terms of what will happen in the real world?

c. The "wavelength" of a wave is the distance a crest of the wave travels in one period. It is also equal to the distance between two adjacent crests. If a tsunami travels at 1200 km/hr, what is its wavelength?

3. **Pebble-in-the-Tire Problem.** As you stop your car at a traffic light, a pebble becomes wedged between the tire treads. When you start off, the distance of the pebble from the pavement varies sinusoidally with the distance you have traveled. The period is, of course, the circumference of the wheel. Assume that the diameter of the wheel is 24 inches.

a. Sketch a graph of this function.
b. Write the particular equation of this function. Be sure to use a *circular* function. It is possible to get a form of the equation that has *zero* horizontal shift.
c. Predict the distance from the pavement when you have gone 15 inches.

4. **Electric Current Problem.** The electricity supplied to your house is called "alternating current" because the current varies sinusoidally with time. See figure 7a below. The frequency of the sinusoid is 60 cycles per second. Suppose that at time t = 0 seconds the current is at its maximum, i = 5 amperes.

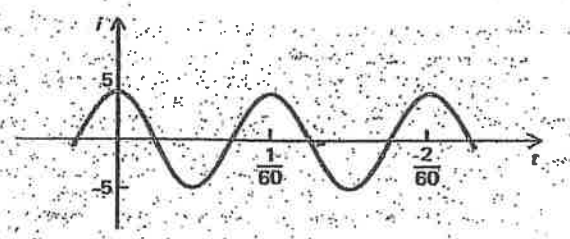


Figure 7a

a. Write an equation expressing current in terms of time.
b. What is the current when t = 0.01?

5. **Roller Coaster Problem.** A portion of a roller coaster track is to be built in the shape of a sinusoid, see Figure 7b above. You have been hired to calculate the lengths of the vertical timber supports to be used.

a. The high and low points on the track are separated by 50 meters horizontally and by 30 meters vertically. The low point is 3 m below the ground. Letting y be the number of meters the track is above the ground and x be the number of meters horizontally from the high point, write the particular equation expressing y in terms of x.
b. How long is the vertical timber at the high point? At x = 4 m? At x = 32 m?

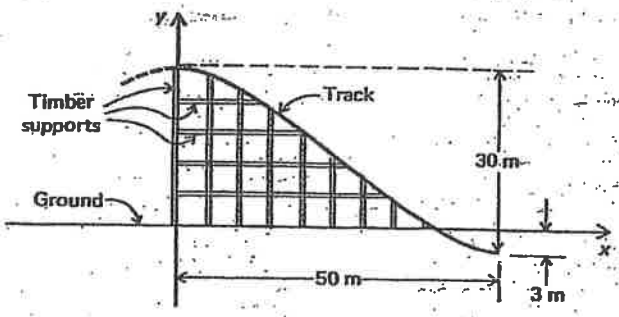
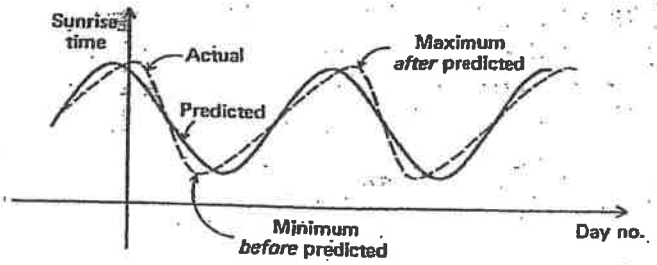


Figure 7b

6. **Sunrise Problem.** Assume that the time of sunrise varies sinusoidally with the day of the year. Let t be the time of day that the Sun rises, and let d be the number of the day of the year, starting with d = 1 on January 1. To calculate the constants in the equation, recall that the period is 365 days. The amplitude and axis location can be calculated from the times of sunrise on the longest and shortest days of the year (ie: June 21 and December 21). You may find these times for various cities in an almanac (they are 5:34 am and 7:24 am CST for San Antonio). The horizontal displacement will, of course be related to the day number on which the sinusoid reaches its maximum, if you use cosine.

a. Sketch a graph of this sinusoid. You may neglect daylight savings time.
b. Write an equation for this function.
c. Calculate the time of sunrise for your city today. Check you answer with today's newspaper to see how close your model is to the actual sunrise time.
d. Predict the time of sunrise on February 16.

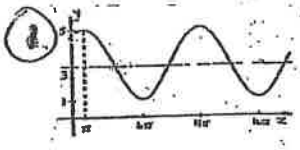


e. If you plot a graph of predicted and actual sunrise times versus d, you will find something like Figure 7c above, where the maximum occurs after the predicted maximum, but the minimum occurs before the predicted minimum. From what you have learned about how the Earth orbits the Sun, think of a reason why the actual sunrise times differ from the predicted ones in this manner.

ANSWERS:

Homework Set #1:

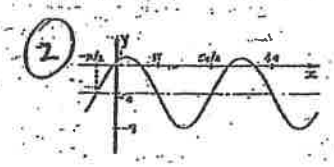
1.



$y = 3 + 2\cos\frac{1}{5}(x - \pi)$

Per. $= \frac{2\pi}{1/5} = 10\pi$

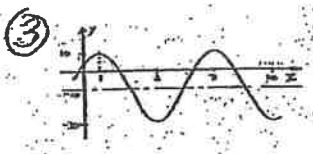
2.



$y = -4 + 5\sin\frac{2}{3}(x + \frac{\pi}{2})$

Per. $= \frac{2\pi}{2/3} = 3\pi$

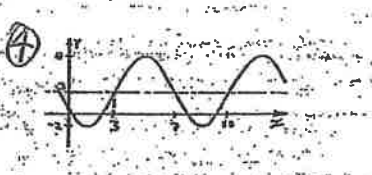
3.



$y = 10 + 20\cos\frac{\pi}{3}(x - 1)$

per. $= 2\pi(\pi/3) = 6$

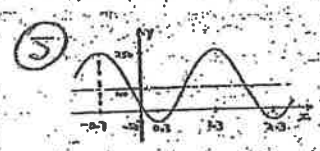
4.



$y = 3 + 5\sin\frac{\pi}{4}(x - 3)$

per. $= 2\pi/(\pi/4) = 8$

5.



$y = 100 + 150\cos\pi(x + 0.7)$
per. $= 2\pi/\pi = 2$

6. $y = 1 + 4\cos(\pi/10)(x - 2)$

Homework Set #2:

1. $y = 5 + 2\cos 2(x - \pi/6)$

2. $y = 4 + 9\cos 8x$

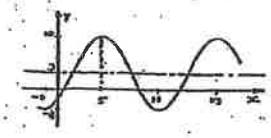
3. $y = 2 + 5\cos(\pi/15)(x + 5)$

4. $y = 0.25 + 0.05\cos(\pi/4)(x + 1)$

5. $z = -8 + 2\cos 5\pi(t + 0.13)$

6. $E = -2.4 + 7.2\cos(\pi/800)(r - 100)$

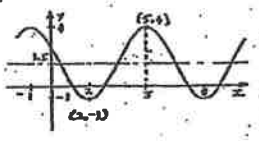
7.



$y = 3 + 7\cos\frac{\pi}{6}(x - 5)$

8. $y = -5 + 8\cos(200\pi)(x - 0.02)$

9.

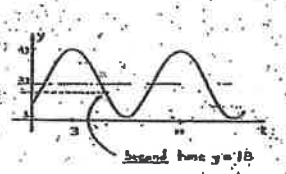


$y = 1.5 + 2.5\cos\frac{\pi}{3}(x - 5)$

10. $S = -8 + 5\cos(\pi/4)(r+5)$ OR
$S = -8 + 5\sin(\pi/4)(r+3)$

Homework Set #3: (1) Ferris Wheel Problem

a.

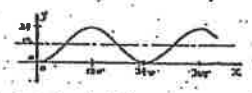


b. The lowest you go is 3 feet above the ground, because seats in a Ferris wheel do not scrape the ground. c. Let y = no. of feet above ground. $y = 23 + 20\cos(\pi/4)(t - 3)$ d. $t = 6 \Rightarrow y = 8.858$ ft. $t = 4\text{-}1/3 \Rightarrow y = 33$ ft. $t = 9 \Rightarrow y = 23$ ft. $t = 0 \Rightarrow y = 8.858$ ft.

(2) Tidal Wave Problem. Let y = no. of meters deep. Let t = no. of min. since tsunami first reached pier. $y = 9 - 10\sin(2\pi/15)t$ a. i. 1.569 m ii. The water has receded beyond the point at which the depth is being measured, meaning that the depth at that point is *zero*. iii. 18.5 m b. According to the model, the minimum depth is −1 meter. So there is a time interval, during which the water is all gone. c. 300 km. d. Since the wave length is so long compared to the amplitude, a person on a ship at sea would not even notice that a tsunami had passed. Note that the motion of the water is up-and-down. It is only the location of the crest of the disturbance which moves horizontally at 1200 km per hour.

(3) Pebble-in-the-Tire Problem

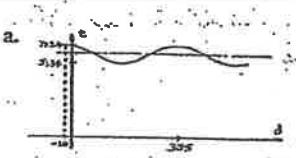
a.



b. Let y = no. of in. from pavement. Let x = no. of in. car travels. $y = 12 - 12\cos(1/12)x$ c. 8.21 in.

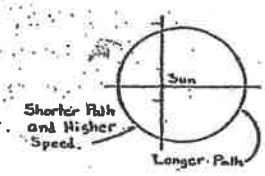
(4) Electric Current Problem a. $i = 5\cos 120\pi t$ b. −4.045 amperes
(5) Roller Coaster Problem $y = 12 + 15\cos(\pi/50)x$ b. i. 27 m ii. 26.53 m iii. 5.61 m
(6) Sunrise Problem

a.



b. $t = 6{:}29 + 55\cos(2\pi/365)(d + 10)$

c. The Earth's orbit is slightly elliptical (see sketch). At the beginning of January, the Earth is *closest* to the Sun. At this time, it has both a *shorter* path and a *higher* speed. So it travels through a quadrant *faster* than predicted by this model.



5 Math III-11 Pre-Calc Introduction Name _____

Review Of Algebra

Please use this packet only as your text book, show all problems and work in your notebook!

PART I. – Algebraic Operations

Section 1.1 – Exponents Review

Properties of Exponents

1. **Product of two powers with equal bases:**

$$x^a \cdot x^b = x^{a+b}$$

2. **Quotient of two powers with equal bases:**

$$\frac{x^a}{x^b} = x^{a-b}$$

3. **Power of a power:**

$$(x^a)^b = x^{ab}$$

4. **Power of a product:**

$$(xy)^a = x^a y^a$$

5. **Power of a quotient:**

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$$

6. **Exponentiation for negative exponents:**

$$x^{-n} = \frac{1}{x^n}$$

7. **Exponentiation for zero exponent:**

$$x^0 = 1, x \neq 0$$

8. **Exponentiation for reciprocal exponents:**

$$x^{1/n} = \sqrt[n]{x}$$

9. **Exponentiation for fractional exponents:**

$$x^{a/b} = \sqrt[b]{x^a} \text{ or } \left(\sqrt[b]{x}\right)^a$$

Example 1

Simplify: $(5x^2y^7)^3(4y^8)$

$= (125x^6y^{21})(4y^8)$ Distribute the exponent 3 to each factor in the product.

$= 500x^6y^{29}$ Multiply the 125 and 4, and the two powers of y.

Example 2

Simplify: $\frac{3^{16}}{3^7}$

$$= \frac{43046721}{2187} = 19683$$

or $3^{16-7} = 3^9$ or 19683

Example 3

Simplify $\sqrt[6]{256} \div \sqrt[4]{64}$

$= 256^{1/6} \div 64^{1/4}$ Transform to exponential form

$= (2^8)^{1/6} \div (2^6)^{1/4}$

$= 2^{4/3} \div 2^{3/2}$ Multiply the exponents & reduce

$= 2^{-1/6}$ Divide by subtracting exponents (with common denominators of course!)

Example 4

Simplify leaving only positive exponents:

$$\frac{\left(6x^{2/7}y^{-4}z^0\right)^3}{9x^2y^5z^{-8}}$$

$= \frac{6^3x^{6/7}y^{-12}z^0}{3^2x^2y^5z^{-8}}$ Distribute the exponent of 3

$= \frac{2^3 3^3 x^{6/7} z^8}{3^2x^2y^{17}}$ Factor 6 and remove negative exponents

$= \frac{24z^8}{x^{8/7}y^{17}}$ Divide constants and powers with equal bases.

Simplify completely leaving only positive exponents. No Calculators!

1. $6x^5 \cdot 3x^{-2}$
2. $5x^{-4} \cdot 2x^{-3}$
3. $3a^{-11} \cdot 17a^5$
4. $(-2x^2)^3(3x^{-1}y^2)^4$
5. $(3x^2y^{-3})^4(2x^{-4}y^5)^3$
6. $(7a^{-5}b^6) \div (21a^4b^{-2})$
7. $\dfrac{5}{a^{-2}} - \dfrac{3}{a^{-1}}$
8. $\dfrac{7}{x^{-5}} - \dfrac{4}{x^{-1}}$
9. $\dfrac{x^{-1}y^{-4}z}{x^{-2}yz^{-3}}$
10. $\dfrac{13x^5y^{-2}z^0}{39xy^{-3}z^2}$
11. $(3758x^{89})^{-53}(3758x^{89})^{53}$
12. $3x^{-1/2} \cdot 4x^{2/3}$
13. $5x^{-1/2} \cdot 6x^{1/8}$
14. $(4x^{-1/2})^3 \div (9x^{1/3})^{-3/2}$
15. $(64x^2)^{-1/6}(32x^5)^{-2/5}$
16. $\dfrac{u^{3.7}p^{4.8}}{u^{-2.9}p^{1.8}}$
17. $\dfrac{d^{-4.3}v^{1.5}}{d^{-3.7}v^{-2}}$
18. $\sqrt[4]{x^{12}y^7}$
19. $\sqrt[5]{m^{30}k^{32}}$

Write your answer in *exact* form, as a fraction if necessary. No Calculators!

20. $64^{2/3}$
21. $64^{5/6}$
22. $64^{-3/2}$
23. $(-64)^{2/3}$
24. $-64^{2/3}$
25. $32^{-6/5}$
26. $128^{2/7}$
27. $81^{-3/2}$
28. $-100^{-3/2}$
29. $\left(\frac{9}{49}\right)^{-3/2}$
30. $\left(\frac{-729}{64}\right)^{-2/3}$

Section 1.2 – Review of Simple Factoring.

Simple Factoring Patterns

Difference of two squares:

$$a^2 - b^2 = (a + b)(a - b)$$

Perfect square trinomials:

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$

Completely Factored Form: A polynomial is completely factored when it is written as a product of two or more *polynomials* with *integer coefficients.*

Sign Patterns for factoring:

If $ax^2 + bx + c$ factors, the signs will be:
(+)(+)
If $ax^2 - bx + c$ factors, the signs will be:
(-)(-)
If $ax^2 + bx - c$ factors, the signs will be:
(+)(-)
If $ax^2 - bx - c$ factors, the signs will be:
(+)(-)

Example 1

Factor completely: $16x^2 - y^2$

You recognize that this is the difference of two perfect squares $(4x)^2 - (y)^2$. Therefore it factors like: $(4x - y)(4x + y)$.

Example 2

Factor completely: $5x^7 - 405x^3$

$= 5x^3(x^4 - 81)$ Factor out common factors.
$= 5x^3(x^2 - 9)(x^2 + 9)$ Difference of 2 squares.
$= 5x^3(x + 3)(x - 3)(x^2 + 9)$ Again!

Example 3

Factor completely: $3x^2 - 16x - 12$

$=$ (+)(-) Decide signs (in either order)

ODDS ONLY

$= (3x + \quad)(x - \quad)$ Only way to factor $3x^2$
$= (3x + 2)(x - 6)$ By trial and error with factors of 12 to get a middle term of -16x.

Factor each of the following completely if possible. Remember to look for a common factor first! No Calculators!

1. $2x^2 + 5x + 3$
2. $3a^2 - 7a + 2$
3. $7t^2 - 10t + 3$
4. $5x^2 - 9x - 2$
5. $2x^2 - x - 3$
6. $6r^2 + 17r + 5$
7. $6p^2 + p - 7$
8. $6u^2 - 11u + 3$
9. $5n^2 - 17n + 12$
10. $10z^2 + 19z + 6$
11. $16x^2 + 18x - 9$
12. $4r^2 - 10r + 6$
13. $8u^2 + 8u + 2$
14. $3x^2 - 10xy + 7y^2$
15. $2a^2 + 11ab + 5b^2$
16. $3r^2 - rt - 2t^2$
17. $2w^2 - 7uw - 15u^2$
18. $16m^2 - 24mn + 9n^2$
19. $14t^2 + 14st - 10s^2$
20. $9b^2 + 6ab - 15a^2$

Section 1.3 – Simplifying Rational Expressions (Products and Quotients)

Recall

$$\frac{x}{a}\cdot\frac{y}{b} = \frac{xy}{ab} \quad \text{and} \quad \frac{a}{b}\div\frac{c}{d} = \frac{a}{b}\cdot\frac{d}{c} = \frac{ad}{bc}$$

A rational expression is **simplified** when its numerator and denominator have no common factors other than 1.

Canceling in a fraction means *dividing* the numerator and denominator of the fraction by the same common factor.

Example 1

Multiply & Simplify

$$\frac{x^2+5x+6}{x^2-x-20}\cdot\frac{x^2+3x-4}{x^2+x-2}$$

$= \frac{(x+2)(x+3)}{(x+4)(x-5)}\cdot\frac{(x+4)(x-1)}{(x-1)(x+2)}$ Factor!

$= \frac{\cancel{(x+2)}(x+3)}{\cancel{(x+4)}(x-5)}\cdot\frac{\cancel{(x+4)}(x-1)}{(x-1)\cancel{(x+2)}}$ Cancel out common factors

$= \frac{x+3}{x-5}$

Example 2

Simplify. $\frac{x-3}{3-x}$

$= \frac{\cancel{x-3}}{(-1)\cancel{(x-3)}}$ Factoring out -1.

$= \frac{1}{-1} = -1$ Cancel out common factor & reduce.

Example 3

Divide and Simplify.

$$\frac{x^2-9}{x^2+x}\div\frac{3-x}{x^2-1}$$

$= \frac{(x+3)(x-3)}{x(x+1)}\cdot\frac{x^2-1}{3-x}$ Factor & change to mult.

$= \frac{(x+3)\cancel{(x-3)}}{x\cancel{(x+1)}}\cdot\frac{\cancel{(x+1)}(x-1)}{(-1)\cancel{(x-3)}}$ Cancel common factors

$= \frac{(x+3)(x-1)}{-x} = \frac{-(x+3)(x-1)}{x}$ Never leave a negative 1 in the denominator!

Simplify Completely! Show All Work! No Calculators!

1. $\frac{2}{x-2}\bullet\frac{x^2-4}{4}$
2. $\frac{x^2+4x+3}{5x}\div\frac{x+1}{x+5}$
3. $\frac{x^2+6x}{6}\bullet\frac{x^3+6}{x^3+6x^2}$
4. $\frac{x+y}{x-y}\div\frac{y+x}{y-x}$
5. $\frac{x^2-49}{49}\bullet\frac{7}{7-x}$
6. $\frac{x^2+2xy+y^2}{x^2-y^2}\div\frac{y+x}{y-x}$
7. $\frac{x^2-3x-4}{3+x}\bullet\frac{x+3}{16-x^2}$

8. $\dfrac{x^2-3xy-10y^2}{xy} \div \dfrac{x^2+7xy+10y^2}{x-5y}$

9. $\dfrac{x^2+x-2}{x^2-4x-12} \bullet \dfrac{x^2-5x-6}{x^2-2x+1}$

10. $\dfrac{x^2-7x+12}{x^2-x-6} \div \dfrac{x^2-16}{x^2+x-2}$

11. $(x^2-5x-14) \cdot \dfrac{x+3}{x^2-4x-21}$

12. $\dfrac{(x+5)(x+8)}{5-x} \div (x+8)$

13. $\dfrac{x+3}{x-7} \div \dfrac{x-7}{x+5} \bullet \dfrac{x+5}{x+3}$

14. $\dfrac{25x^2-1}{9x^2-4y^2} \div \dfrac{5x-1}{3x-2y} \div \dfrac{5x+1}{3x+2y}$

15. $\dfrac{x^4-y^4}{x^3-y^3} \bullet \dfrac{x^2-y^2}{x^3+y^3} \bullet \dfrac{x^6-y^6}{x^2+y^2}$

Section 1.4 – Simplifying Rational Expressions (Sums and Differences)

Recall

To add or subtract fractions, you must have common denominators. It will save you a great deal of time if you use the least common multiple (LCM) of denominators as the common denominator.

For Example: $\frac{2}{3}+\frac{3}{7}-\frac{2}{9}$ A common denominator is: (3)(7)(9) or 189 but the LCM is 63, using 63 will make it easier.

$\frac{2(21)+3(9)-2(7)}{63} = \frac{55}{63}$ which is in lowest terms!

Example 1

Subtract & simplify: $\frac{7x-1}{x^2-2x-3} - \frac{6x}{x^2-x-2}$

$= \frac{7x-1}{(x-3)(x+1)} - \frac{6x}{(x-2)(x+1)}$ Factor first!

$= \frac{(7x-1)(x-2)-(6x)(x-3)}{(x-3)(x+1)(x-2)}$ Combine with LCM as denominator.

$= \frac{7x^2-15x+2-(6x^2-18x)}{(x-3)(x+1)(x-2)}$ Don't forget to distribute the – sign!

$= \frac{7x^2-15x+2-6x^2+18x}{(x-3)(x+1)(x-2)}$ Now simplify.

$= \frac{x^2+3x+2}{(x-3)(x+1)(x-2)} = \frac{(x+2)\cancel{(x+1)}}{(x-3)\cancel{(x+1)}(x-2)}$ Reduce!

$= \frac{(x+2)}{(x-3)(x-2)}$

Example 2

Add & simplify. $\frac{3}{x-2}+\frac{4}{2-x}+\frac{1}{x}$

The second fraction you can factor a -1 from the denominator then change the + sign to a – sign.

$$\frac{4}{2-x} = \frac{4}{(-1)(x-2)} = \frac{-4}{x-2}$$

Then our fractions with a common denominator are:

$= \frac{3}{x-2} - \frac{4}{x-2} + \frac{1}{x} = \frac{3x-4x+1(x-2)}{(x-2)x} = \frac{3x-4x+x-2}{x(x-2)}$

$= \frac{-2}{x(x-2)}$ Simplify!

Example 3

Simplify. $\frac{x-5+\frac{6}{x}}{1-\frac{9}{x^2}} = \frac{\frac{x^2-5x+6}{x}}{\frac{x^2-9}{x^2}}$ Find a common denominator for the larger numerator and denominator. Then factor and change the large fraction to a multiplication problem.

$= \frac{(x-2)\cancel{(x-3)}}{\cancel{x}} \cdot \frac{x^{\cancel{2}}}{\cancel{(x-3)}(x+3)}$ And reduce!

$= \frac{x(x-2)}{(x+3)}$

ODDS ONLY

Examples **Simplify Completely! Show All Work!** **No Calculators!**

E1: $\frac{2x-1}{3}-\frac{4x-8}{6}$

E2: $\frac{x}{x+y}+\frac{y}{x-y}$

E3: $\frac{3}{x-1}+\frac{1}{1-x}$

E4: $\frac{4x}{(x+y)^2}-\frac{4}{x+y}$

E5: $\frac{3}{x+5}-\frac{2x-20}{x^2-25}$

Section 1.4: Simplify Completely! Show All Work! **No Calculators!**

1. $\frac{x-3}{3}-\frac{x-4}{4}$
2. $\frac{1}{x+1}+\frac{1}{x-1}$
3. $\frac{6}{2x-3y}-\frac{3}{3y-2x}$
4. $\frac{2x-1}{x+1}-\frac{2x-1}{x-1}$
5. $\frac{a-b}{c-d}-\frac{b-a}{d-c}$
6. $\frac{1}{y-x}+\frac{x}{(x-y)^2}$
7. $\frac{2}{x-4}-\frac{x+12}{x^2-16}$
8. $\frac{x-y}{x^2-y^2}+\frac{1}{2x+3y}$
9. $\frac{x^2+5x+4}{x+4}-\frac{x^2-5x+6}{x-2}$
10. $x+2-\frac{x^2+x-6}{x-3}$
11. $\frac{x+a}{x-a}-\frac{x^2+a^2}{ax-a^2}$
12. $\frac{1}{x+1}-\left(\frac{1}{x-1}-\frac{1}{x^2-1}\right)$
13. $\frac{1}{x-1}-\frac{2}{x-2}+\frac{1}{x-3}$
14. $\frac{3}{x^2+x-2}-\frac{5}{x^2-x-6}$
15. $\frac{3x+13}{x^2-3x-10}-\frac{16}{x^2-6x+5}$
16. $\frac{x-2}{x^2-x-2}+\frac{x-4}{x^2-5x+4}$

Every Other Odd (EOO)

Factoring sum & difference of cubes

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Factoring by Splitting the Middle Term

To factor $ax^2 + bx + c$

1. Multiply a by c.
2. Look for two factors of the answer whose sum is b. (Or start with numbers that add up to b and find two whose product is ac.)
3. Split the middle term into terms with these numbers as coefficients.

Example 1

Factor $20x^2 + 39x + 18$ by splitting the middle term.

$(20)(18) = 360$ Look for factors of 360 that add up to 39 (coefficient of middle term) Aha! 15 & 24

Therefore, $20x^2 + 24x + 15x + 18$

$= 4x(5x + 6) + 3(5x + 6)$

$= (5x + 6)(4x + 3)$

Example 2

Factor $x^6 - 343$

Can you recognize the perfect cubes? $(x^2)^3 - 7^3$
$= (x^2 - 7)(x^4 + 7x^2 + 49)$

E4: Factor: $a^3 - 1$

E5: Factor: $8x^3 + 27$

Section 1.5 – Factor each of the following completely if possible. Remember to look for a common factor first! No Calculators!

1. $a^2 + 3a + ab + 3b$
2. $8x^2 + 12xy + 10xz + 15yz$
3. $8x^2 - 3x - 8x + 3$
4. $x^2 + 6x + 9 - y^2$
5. $x^2 - 10x + 25 - 9a^2$
6. $3sx + 15s + x^2 + 3x - 10$
7. $6x^2 + 25x + 25$
8. $3x^2 + 20x + 40$
9. $2m^2 + 15m - 50$
10. $8x^5 + 2x^4 - 3x^3$
11. $27ru^2 + 36ru + 12r$
12. $12(x + 3)x^2 - 2(x + 3)x - 4(x + 3)$
13. $12x^3 + 45x^2 + 32x + 120$
14. $18x^3 - 27x^2 + 8x - 12$
15. $7r^2s - 6r^2 - 7s + 6$
16. $10x^2 - 29x - 21$
17. $8x^2 + 49x + 6$
18. $x^2 + 16x + 64$
19. $x^2 - 9$
20. $36x^2 - 144$
21. $36 - (x - 5)^2$
22. $30x^2 + 41x - 6$
23. $x^2 - 21x + 68$
24. $18x^2 - 15x + 2$
25. $18x^2 - 13x + 3$
26. $4x^2 - 16y^2$
27. $100 - (x - y)^2$
28. $27a^3 + 64y^3$

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Section 1.5B – More Factoring

1. $3a^2 + 3a - 6$
2. $81x^2 + 36xy + 4y^2$
3. $c^2 - 4d^2$
4. $8x^3 + 343y^3$
5. $64x^6y^3 - 27z^3$
6. $z^2 - (3x + 2y)^2$
7. $12x^2 - 11xy - 56y^2$
8. $x^8 - 2x^4y^4 - 15y^8$
9. $5x^2 - 3x + 17$
10. $x^2 - ax + bx - ab$

11. $a^3 + b^3 + a^2 - b^2$ 12. $25x^2 - 10x + 1 - 4y^2$

13. $a^2 + 4b^2 - 25c^2 + 4ab$

14. $2a^2b - 16ab + 32b$ 15. $16x^4 + 4x^2 - 2$

16. $x^8 - 16$ 17. $ax^2 - a + bx^2 - b$

18. $-5x^4 + 2000$ 19. $6(x+y)^2 - 5z(x+y) + z^2$

20. $y^4 - 13y^2 + 36$ 21. $15c^2 - 34c - 16$

22. $x^3 - 4x^2 + 3x - 2$ 23. $18x^3 + 9x^2 - 2x - 1$

24. $125x^3 - y^6$ 25. $a - a^3$

26. $12x^2 + 35x + 8$ 27. $2x^3 - 16y^3$

28. $3x^2 - 2x + 11$ 29. $(x+2)^2 - (y-1)^2$

30. $2x^3 + 3x^2 - 3x - 2$

Section 1.6 – Review of Logarithmic Properties
Remember the properties of Logs?

Log form: $\log_{base} \# = exp$
Exp form: $base^{exp} = \#$
Product Property: $\log_b MN = \log_b M + \log_b N$
Quotient Property: $\log_b \frac{M}{N} = \log_b M - \log_b N$
Power Property: $\log_b M^k = k \log_b M$
Change of Base: $\log_b M = \frac{\log_c M}{\log_c b}$

Example 1

Write as a single logarithm. $\log_5 12 - 3\log_5 3$

$= \log_5 12 - \log_5 3^3$ Using the Power Property
$= \log_5 12/3^3$ Using the Quotient Property
$= \log_5 4/9$ Simplify!

Example 2

Write $\log_5 27$ using base 10.

$= \frac{\log 27}{\log 5}$ This uses change of base. You cannot go any farther without a calculator (or log chart).

Write each expression as a single logarithm.

E1: $\log_7 3 + 2\log_7 5$

E2: $\frac{1}{2}\log_7 25 - 3\log_7 5$

E3: $\log_7 5$ Write in base 10. (Hint: Use change of base)

Section 1.6 – Review of Logarithmic Properties

Write each expression as a single logarithm or simplify completely. **No Calculators!**

ODDS only

1. $\log_3 5 + \log_3 7$ 2. $\log_2 24 - \log_2 8$

3. $\log_7 2 + \log_7 5 + \log_7 3$

4. $\log_5 48 - \log_5 12 + \log_5 4$

5. $5\log_{12} 2$ 6. $3\log_6 15 - \log_6 25$

7. $\log_{49} 49^8$ 8. $3\log_{24} 24^{-2/3}$

9. $3^{\log_{10} 9}$ 10. $10^{\log_{10} 2001}$

11. $\ln x + \ln 2$ 12. $\log_4 z - \log_4 y$

13. $2\log_2 (x+4)$ 14. $\ln x - 3\ln(x+1)$

15. $\ln x - 2[\ln(x+2) + \ln(x-2)]$

16. $\frac{1}{3}[2\ln(x+3) + \ln x - \ln(x^2 - 1)]$

17. $\frac{1}{3}[\ln y + 2\ln(y+4)] - \ln(y-1)$

18. $2\ln 3 - \frac{1}{2}\ln(x^2+1)$

19. $\log_3 9$

20. $\log_4 16^{1.2}$

21. $\ln e^{4.5}$

Use the properties of logs to write the expression as a sum, difference, and/or multiple of logs. No Calculators!

22. $\log_{10} 5x$

23. $\log_{10} \frac{1}{x}$

24. $\log_8 x^4$

25. $\ln \sqrt{z}$

26. $\ln xyz$

27. $\ln \sqrt{a-1}$

28. $\ln z(z-1)^2$

29. $\ln \sqrt[3]{\frac{x}{y}}$

30. $\ln \frac{x^4\sqrt{y}}{z^5}$

****

Write the expression as a single logarithm of a single argument.

31. $\log_a x + \log_a y - \log_a m$

32. $(\log_b k - \log_b m) - \log_b a$

33. $2\log_m a - 3\log_m (b^2)$

34. $\frac{1}{2}\log_y (p^3q^4) - \frac{2}{3}\log_y (p^4q^3)$

35. $2\log_a (z-1) + \log_a (3z+2),\ z > 1$

36. $\log_b (2y+5) - \frac{1}{2}\log_b (y+3)$

37. $\ln y + \ln z$

38. $\frac{1}{3}\log_3 5x$

39. $\ln(x-2) - \ln(x+2)$

40. $\log_5 8 - \log_5 t$

41. $\frac{1}{2}[\ln(x+1) + 2\ln(x-1)] + 3\ln x$

42. $-4\log_6 (2x)$

43. $\frac{3}{2}\log_7 (z-2)$

44. $\frac{3}{2}\ln 5t^6 - \frac{3}{4}\ln t^4$

Find the exact value of the logarithm.

45. $\log_6 \sqrt[3]{6}$

46. $\log_5 \left(\frac{1}{125}\right)$

47. $\ln \sqrt[4]{e^3}$

Use the properties of logarithms to write the expression as a sum, difference, and/or multiple of logarithms.

48. $\log_b \frac{x^2}{y^2z^3}$

49. $\log 10z$

50. $\log \frac{y}{2}$

51. $\log_6 z^{-3}$

52. $\ln \sqrt[3]{t}$

53. $\ln \frac{xy}{z}$

54. $\ln \left(\frac{x^2-1}{x^3}\right)$

55. $\ln \sqrt{\frac{x^2}{y^3}}$

56. $\ln \frac{x}{\sqrt{x^2+1}}$

57. $\ln \sqrt{x^2(x+2)}$

58. $\log_b \frac{\sqrt{x}y^4}{z^4}$

Section 1.7 – Review of Synthetic Division

Example 1

Use synthetic division to divide:

$$\frac{3x^4 - 19x^3 - 21x^2 + 51x - 14}{x-2}$$

2	3	−19	−21	51	−14

You synthetically divide by the root of (x – 2) or the value of x that makes (x – 2) = 0. That is 2. Write the coefficients of the polynomial in decreasing order. Be sure to use a 0 for any variable "missing" in decreasing order.

Then, bring down the first coefficient, 3. Multiply 3 by the 2 and place the product below the -19. Add down. Continue in the same manner until the entire block is filled as below.

2	3	−19	−21	51	−14
		6	−26	−94	−86
*	3	-13	-47	-43	-100

Therefore, -100 is the remainder and the polynomial part of the answer begins 1 degree lower than the original polynomial or:

$3x^3 - 13x^2 - 47x - 43 - 100/(x-2)$

E1: Divide $x^3 + 4x^2 - 3x - 5$ by $(x + 3)$ using synthetic division. Write your remainder as a fraction.

Section 1.7 – Divide each of the following using synthetic division. Write your remainder as a fraction. <u>No Calculators!</u>

1. $(x^3 - 5x^2 + 2x - 2) \div (x - 1)$
2. $(3x^3 - 8x^2 - 2x - 3) \div (x - 3)$
3. $(-2x^4 + 15x^2 - 6x + 2) \div (x + 3)$
4. $(x^4 - 12 + x^2 - x) \div (x - 2)$
5. $(x^4 - 1) \div (x + 1)$ 6. $(4 - x^6) \div (x - 1)$

Simplify. <u>No Calculators!</u>

7. $\frac{4x^3-8x^2+x+3}{2x-3}$ 8. $\frac{x^3+3x^2-x-3}{x+1}$

9. $\frac{x^4+6x^3+11x^2+6x}{x^2+3x+2}$ 10. $\frac{x^4+9x^3-5x^2-36x+4}{x^2-4}$

Section 1.8 – Simplifying Radical Expressions

An expression is in **simplest radical form** if

1. The radicand of the nth root contains no n^{th} powers as factors,
2. The root index is as low as possible, and
3. there are no radicals in the denominator.

<u>Example 1</u>

Simplify. $\frac{2}{5+\sqrt{3}}$

Multiplying by one in the form of $\frac{\sqrt{3}}{\sqrt{3}}$ will NOT remove the square root from the denominator, can you see that? Here we must multiply by one in the form of the conjugate or the denominator with the opposite sign in between the terms (ie: $5-\sqrt{3}$)

$= \frac{2}{5+\sqrt{3}} \cdot \frac{5-\sqrt{3}}{5-\sqrt{3}} = \frac{2(5-\sqrt{3})}{25-5\sqrt{3}+5\sqrt{3}-3}$ Now distribute and simplify.

$= \frac{10-2\sqrt{3}}{22} = \frac{2(5-\sqrt{3})}{22} = \frac{5-\sqrt{3}}{11}$

<u>Example 2</u>

Simplify. $\frac{2+\frac{1}{\sqrt{3}}}{2-\frac{1}{\sqrt{3}}}$

Get a common denominator in the upper & lower fractions.

$\frac{2+\frac{1}{\sqrt{3}}}{2-\frac{1}{\sqrt{3}}} = \frac{\frac{2\sqrt{3}+1}{\sqrt{3}}}{\frac{2\sqrt{3}-1}{\sqrt{3}}} = \frac{2\sqrt{3}+1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2\sqrt{3}-1}$

$= \frac{2\sqrt{3}+1}{2\sqrt{3}-1}$ Now simplify using conjugate.

$= \frac{2\sqrt{3}+1}{2\sqrt{3}-1} \cdot \frac{2\sqrt{3}+1}{2\sqrt{3}+1} = \frac{12+4\sqrt{3}+1}{12-1} = \frac{13+4\sqrt{3}}{11}$

<u>Example 3</u>

Simplify. $\frac{1}{2+\sqrt{5}-\sqrt{3}}$

Regroup to isolate one root. $\frac{1}{(2+\sqrt{5})-\sqrt{3}}$

Multiply by conjugate. $\frac{1}{(2+\sqrt{5})-\sqrt{3}} \cdot \frac{(2+\sqrt{5})+\sqrt{3}}{(2+\sqrt{5})+\sqrt{3}}$

$= \frac{(2+\sqrt{5})+\sqrt{3}}{(2+\sqrt{5})^2-3}$ or $\frac{2+\sqrt{5}+\sqrt{3}}{6+4\sqrt{5}}$ after simplifying!

Now you need to use the conjugate again to remove the root from the denominator!

$= \frac{2+\sqrt{5}+\sqrt{3}}{6+4\sqrt{5}} \cdot \frac{6-4\sqrt{5}}{6-4\sqrt{5}} = \frac{(2+\sqrt{5}+\sqrt{3})(6-4\sqrt{5})}{36-80}$ And simplify to get:

$= \frac{-8-2\sqrt{5}+6\sqrt{3}-4\sqrt{15}}{-44} = \frac{(-2)(4+\sqrt{5}-3\sqrt{3}+2\sqrt{15})}{(-2)(22)}$

$= \frac{4+\sqrt{5}-3\sqrt{3}+2\sqrt{15}}{22}$ Finished!

Simplify each expression showing all work. <u>No Calculators!</u>

E1: $\frac{2}{3\sqrt{2}} + \frac{3\sqrt{2}}{6}$

E2: $\left(\sqrt{5}+\sqrt{3}\right)^2$

E3: $\frac{1}{\sqrt{7}-2}$

E4: $\frac{5}{\sqrt{5}+\sqrt{3}}$

Section 1.8 – Simplifying Radical Expressions

Simplify each expression showing all work. <u>No Calculators!</u>

1. $\frac{3}{2\sqrt{2}} + \frac{5\sqrt{2}}{4}$
2. $\frac{7}{5\sqrt{3}} - \frac{8\sqrt{3}}{15}$
3. $\frac{12}{\sqrt{6}} + \sqrt{6}$
4. $4\sqrt{5} - \frac{15}{\sqrt{5}}$
5. $7\sqrt{3} - \frac{12}{\sqrt{3}} + \sqrt{75}$
6. $4\sqrt{5} + \frac{35}{\sqrt{5}} - \sqrt{125}$
7. $\left(\sqrt{7}+\sqrt{2}\right)^2$
8. $\left(\sqrt{5}-\sqrt{3}\right)^2$
9. $\left(2\sqrt{5}-3\right)^2$
10. $\left(4+3\sqrt{6}\right)^2$

Every Other Odd for 1-69

11. $(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})$

12. $(\sqrt{7}-2)(\sqrt{7}+2)$

13. $(2\sqrt{3}-\sqrt{2})(2\sqrt{3}+\sqrt{2})$

14. $(3\sqrt{5}+\sqrt{7})(3\sqrt{5}-\sqrt{7})$

15. $(6\sqrt{7}+\sqrt{15})(\sqrt{7}-\sqrt{3})$

16. $(\sqrt{12}-\sqrt{6})(\sqrt{3}+\sqrt{27})$

17. $\dfrac{1}{\sqrt{5}-1}$

18. $\dfrac{4}{3-2\sqrt{2}}$

19. $\dfrac{4}{\sqrt{7}+\sqrt{3}}$

20. $\dfrac{57}{5\sqrt{3}-3\sqrt{2}}$

21. $\dfrac{\sqrt{3}-1}{\sqrt{2}-1}$

22. $\dfrac{3\sqrt{3}-1}{3\sqrt{2}-1}$

23. $\dfrac{7\sqrt{2}+3}{7\sqrt{2}-3}$

24. $\dfrac{4\sqrt{7}+3\sqrt{2}}{5\sqrt{2}+2\sqrt{7}}$

25. $\dfrac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$

26. $\dfrac{1-\frac{1}{\sqrt{5}}}{1+\frac{1}{\sqrt{5}}}$

27. $\dfrac{12}{2+\sqrt{3}-\sqrt{7}}$

28. $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}-1}$

29. $\dfrac{1}{\sqrt{3}+\sqrt{2}-\sqrt{5}}$

30. $\dfrac{1}{\sqrt{5}+\sqrt{3}+2\sqrt{2}}$

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Simplify each expression showing all work. <u>No Calculators!</u>

31. $2\sqrt{5}+7\sqrt{5}$

32. $8\sqrt{7}-3\sqrt{7}$

33. $13\sqrt{6}-\sqrt{6}$

34. $\sqrt{12}+5\sqrt{3}$

35. $7\sqrt{5}+\sqrt{45}$

36. $\sqrt{18}+\sqrt{50}$

37. $6\sqrt{12}-\sqrt{75}$

38. $3\sqrt{7}-\sqrt{175}$

39. $3\sqrt{5}-\sqrt{80}+\sqrt{20}$

40. $\sqrt{50}-\sqrt{18}-2\sqrt{12}$

41. $\frac{\sqrt{3}}{\sqrt{5}}\cdot\frac{\sqrt{15}}{\sqrt{8}}$

42. $\frac{\sqrt{18}}{\sqrt{7}}\cdot\sqrt{\frac{14}{5}}$

43. $\frac{\sqrt{30}}{\sqrt{14}}\div\frac{\sqrt{15}}{\sqrt{32}}$

44. $\sqrt{\frac{27}{5}}\div\frac{\sqrt{10}}{\sqrt{3}}$

45. $\frac{3}{\sqrt{3}}+\frac{18}{\sqrt{27}}$

46. $\sqrt{45}-\frac{10}{\sqrt{5}}$

47. $6\sqrt{\frac{2}{3}}+6\sqrt{\frac{3}{2}}$

48. $3\sqrt{3}-\sqrt{48}+3\sqrt{\frac{1}{3}}$

49. $6\sqrt{\frac{7}{4}}-14\sqrt{\frac{1}{7}}+3\sqrt{28}$

50. $\sqrt{6}(\sqrt{3}-\frac{1}{\sqrt{3}})$

51. $(\sqrt{6}-2\sqrt{3})(3\sqrt{6}+\sqrt{3})$

52. $(2\sqrt{15}-4\sqrt{5})(6\sqrt{15}-3\sqrt{5})$

53. $(\sqrt{3}-7)^2$

54. $(\sqrt{6}+\sqrt{3})^2$

55. $(5\sqrt{2}-3\sqrt{7})^2$

56. $(\sqrt{5}+3)(\sqrt{5}-3)$

57. $(\sqrt{13}-\sqrt{2})(\sqrt{13}+\sqrt{2})$

58. $(5\sqrt{3}-1)(5\sqrt{3}+1)$

59. $(3\sqrt{2}+5\sqrt{3})(3\sqrt{2}-5\sqrt{3})$

60. $(\sqrt{1001}+\sqrt{2001})(\sqrt{1001}-\sqrt{2001})$

61. $\frac{20}{\sqrt{6}+2}$

62. $\frac{24}{\sqrt{15}-3}$

63. $\frac{26}{4-\sqrt{3}}$

64. $\frac{45}{\sqrt{2}+\sqrt{7}}$

65. $\frac{48}{\sqrt{11}-\sqrt{5}}$

66. $\frac{\sqrt{13}+3}{5-\sqrt{13}}$

67. $\frac{\sqrt{3}+6}{\sqrt{3}+5}$

68. $\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

69. $\frac{\sqrt{10}-\sqrt{6}}{\sqrt{10}+\sqrt{6}}$

70. $\frac{\sqrt{1776}+\sqrt{1066}}{\sqrt{1776}-\sqrt{1066}}$

PART II. – Solving Equations

Section 2.1 – Solving quadratic equations. Solve each equation showing all work. No Calculators!

Quadratic Equations may be solved by:
1. Factoring
2. Using the Quadratic Formula $\left(x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\right)$
3. Completing the Square. ("a" must be 1)

Example 1

Solve. $x^2 + 15x - 34 = 0$

Factor! $(x + 17)(x - 2) = 0$ therefore,
$x = -17$ or $x = 2$

Example 2

Solve. $x^2 - 10x + 34 = 0$

Doesn't factor, let's try the Quadratic Formula!

$$x = \frac{10 \pm \sqrt{100-(4)(1)(34)}}{2(1)} = \frac{10 \pm \sqrt{-36}}{2} = \frac{10 \pm 6i}{2}$$

$x = (10+6i)/2$ or $5+3i$ and $x = (10-6i)/2$ or $5-3i$

Example 3

Solve. $x^2 + 8x + 25 = 5x + 6$

First set equal to zero.

$x^2 + 3x + 19 = 0$ Let's complete the square!

$x^2 + 3x + \qquad = -19$

To decide what to add to both sides to complete the square: take the "b" (3 for this problem), divide it by 2 and square the quantity. Or:

$$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

Therefore,

$x^2 + 3x + \frac{9}{4} = -19 + \frac{9}{4}$ Complete the square!

$\left(x + \frac{3}{2}\right)^2 = \frac{-67}{4}$ Take the square root of both sides

$x + \frac{3}{2} = \pm\frac{i\sqrt{67}}{2}$ and $x = -\frac{3}{2} \pm \frac{i\sqrt{67}}{2}$

E1: $5a^2 + 7a + 2 = 0$

E2: $2x^2 + 5x + 4 = 0$

Section 2.1 – Solve each equation showing all work. No Calculators!

E00

1. $2n^2 - 5n + 3 = 0$
2. $5z^2 + 9z - 2 = 0$
3. $6y^2 - 11y + 3 = 0$
4. $4r^2 + 4r + 1 = 0$
5. $12x^2 - x - 6 = 0$
6. $x^2 + 8x + 16 = 25$
7. $6y^2 + 13y = -5$
8. $2a(a + 1) + a^2 = 2a + 12$
9. $3x^2 + 2x = 48 + 2x$
10. $-45 + 6w = w^2 - 36$
11. $4x^2 - 4x = 5x + 2 - x^2$
12. $p^2 - 3p - 88 = 0$
13. $d^2 - \frac{3}{4}d + \frac{1}{8} = 0$
14. $t^2 - 3t - 7 = 0$
15. $s^2 - 5s + 9 = 0$
16. $4x^2 - 2x + 9 = 0$
17. $2k^2 + 5k = 9$
18. $x^2 - 3x - 28 = 0$
19. $4r^2 - r = 5$
20. $x^2 - 2\sqrt{6}\,x - 2 = 0$

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21. $-25x - 6 = -9x^2$
22. $16r^2 = 25$
23. $10 = 11y - 3y^2$
24. $3s^2 + 6s - 45 = 0$
25. $3a^2 - 12a + 12 = 0$
26. $6y^2 + 15y - 9 = 0$
27. $10m^2 - 4m - 14 = 0$
28. $a(2a + 1) + 2a^2 = a + 16$
29. $3m^2 - 3m = 3 + 10m - 7m^2$
30. $3s^2 - 5s + 9 = 0$
31. $p^2 + 2p + 8 = 0$
32. $4w^2 + 19w - 5 = 0$
33. $z^2 - 2z - 24 = 0$
34. $3g^2 - 12g = -4$
35. $x^2 - 10x + 21 = 0$
36. $6m^2 + 7m - 3 = 0$
37. $36d^2 - 84d + 49 = 0$
38. $3p^2 + 4p = 8$
39. $54b = -9b^2 - 45$
40. $36z^2 + 48z = 9$
41. $8y^2 - y(y - 1) = 0$
42. $7x(x + 2) = 2x^2$
43. $5t^2 - 10t = -5$
44. $7y^2 + 42y = -63$
45. $2z - z^2 = -6z + 16$
46. $14n^2 - 5 = 16 - 7n$
47. $3r^2 + 24 = 18r$
48. $15b^2 + 10b - 25 = 0$
49. $4r^2 - 2r - 6 = 0$
50. $5x^2 - 30x + 45 = 0$

Section 2.2 – Solving rational equations.

Solving Rational Equations:

1. Write the domain (or decide what x cannot be),
2. Multiply both sides of the equation by the smallest expression (LCM) needed to eliminate all of the fractions,
3. Solve the resulting equation,
4. Discard any extraneous solutions, and
5. Write your solution(s).

Example 1

Solve. $\frac{2}{y+2} + \frac{3}{y} = \frac{-y}{y+2}$

Decide what values of y will make any denominators zero. (-2 and 0) Eliminate these at the end if necessary. The LCM of all the denominators is (y+2)(y). Therefore, that is what we multiply all the way through by, making sure we use every term!

$$\frac{2}{\cancel{y+2}}(y)\cancel{(y+2)} + \frac{3}{\cancel{y}}\cancel{(y)}(y+2) = \frac{-y}{\cancel{y+2}}(y)\cancel{(y+2)}$$

By canceling common factors, our denominators will be gone to yield:

$2y + 3(y+2) = -y(y)$ Then solve.
$2y + 3y + 6 = -y^2$
$y^2 + 5y + 6 = 0$ Factor
$(y+2)(y+3) = 0$
Therefore $y = -2$ or $y = -3$.
y cannot be -2 therefore $y = -3$ is our only solution.

Example 2

Solve. $3 - \frac{22}{x+5} = \frac{6x-1}{2x+7}$
First, x cannot be -5 or -7/2. (Denominators would be zero!)

Multiply all the way through by (x+5)(2x+7) our LCM of the denominators.

$$3(x+5)(2x+7) - \frac{22}{\cancel{x+5}}\cancel{(x+5)}(2x+7) = \frac{6x-1}{\cancel{2x+7}}(x+5)\cancel{(2x+7)}$$

Then simplify:

$3(2x^2 + 17x + 35) - 22(2x+7) = (6x-1)(x+5)$
$6x^2 + 51x + 105 - 44x - 154 = 6x^2 + 29x - 5$
$-22x - 49 = -5$
$-22x = 44$
$x = -2$ which is our only solution.

EOO

Solve each equation showing all work. No Calculators!

E1: $\frac{12}{x}+x-8=0$

E2: $a+\frac{a^2-5}{a^2-1}=\frac{a^2+a+2}{a+1}$

Section 2.2 – Solve each equation showing all work. No Calculators!

1. $\frac{1}{m}=\frac{m-34}{2m^2}$

2. $\frac{10}{n^2-1}+\frac{2n-5}{n-1}=\frac{2n+5}{n+1}$

3. $\frac{7a}{3a+3}-\frac{5}{4a-4}=\frac{3a}{2a+2}$

4. $\frac{2q}{2q+3}-\frac{2q}{2q-3}=1$

5. $\frac{-4}{x-1}=\frac{7}{2-x}+\frac{3}{x+1}$

6. $x+\frac{x}{x-2}=\frac{2}{x-2}$

7. $\frac{x}{x-3}-\frac{7}{x+5}=\frac{24}{x^2+2x-15}$

8. $\frac{3x}{x+4}+\frac{4x}{x-3}=\frac{84}{x^2+x-12}$

9. $\frac{3}{x-3}+\frac{4}{x-4}=\frac{25}{x^2-7x+12}$

10. $\frac{x+2}{x-3}+\frac{x-2}{x-6}=2$

11. $\frac{2}{x+2}-\frac{x}{2-x}=\frac{x^2+4}{x^2-4}$

12. $\frac{1}{1-x}=1-\frac{x}{x-1}$

13. $\frac{x+3}{2x-3}=\frac{18x}{4x^2-9}$

14. $\frac{4x}{x^2-9}-\frac{x-1}{x^2-6x+9}=\frac{2}{x+3}$

15. $\frac{3x}{x-2}+\frac{2x}{x+3}=\frac{30}{x^2+x-6}$

16. $\frac{x^3+3x-9}{x(x-3)}-\frac{x+6}{x-3}=\frac{3}{x}$

17. $\frac{11x}{x+20}+\frac{24}{x}=11+\frac{88}{x(x+20)}$

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18. $\frac{2}{y+2}+\frac{3}{y}=\frac{-y}{y+2}$

19. $\frac{1}{b+2}+\frac{1}{b+2}=\frac{3}{b+1}$

20. $1=\frac{1}{1-a}+\frac{a}{a-1}$

21. $\frac{1}{3m}+\frac{6m-9}{3m}=\frac{3m-3}{4m}$

22. $x+\frac{2x}{x-1}=\frac{3-x}{x-1}$

23. $\frac{x}{x+2}+\frac{7}{x-5}=\frac{14}{x^2-3x-10}$

24. $\frac{4x}{x-1}-\frac{5x}{x-2}=\frac{2}{x^2-3x+2}$

25. $\frac{11x}{x+20}+\frac{24}{x}=11+\frac{88}{x(x+20)}$

26. $\frac{3x+2}{x-1}+\frac{2x-4}{x+2}=5$

27. $\frac{x}{x+4}-\frac{4}{x-4}=\frac{x^2+16}{x^2-16}$

28. $\frac{x}{x-1}-\frac{2}{1-x^2}=\frac{8}{x+1}$

29. $3-\frac{22}{x+5}=\frac{6x-1}{2x+7}$

30. $\frac{x}{x^2-2x+1}=\frac{2}{x+1}+\frac{4}{x^2-1}$

31. $\frac{5}{x-6}-\frac{4}{x+3}=\frac{x+39}{x^2-3x-18}$

32. $\frac{5x}{x-5}+\frac{4}{x+6}=\frac{54x+5}{x^2+x-30}$

Section 2.3 – Solving radical equations.

A **radical equation** is an equation in which a variable appears under a radical sign.

Example 1

Solve. $3 = x + \sqrt{x-1}$

First: Get the radical on one side of the equation by itself. Like:

$3 - x = \sqrt{x-1}$ Then square both sides of the equation.

$(3-x)^2 = (\sqrt{x-1})^2$ Simplify!
$9 - 6x + x^2 = x - 1$
$x^2 - 7x + 10 = 0$ Solve.
$(x-5)(x-2) = 0$
Therefore $x = 5$ or $x = 2$. Substitute these into your original equation (YES! ALWAYS!) to check:

$x = 2$	$x=5$
$3 = 2 + \sqrt{2-1}$	$3 = 5 + \sqrt{5-1}$
$3 = 2 + 1$	$3 = 5 + 2$
$3=3$	$3 \neq 7$

Therefore only 2 is a solution!

Example 2

Solve $\sqrt{x^2+5x-6} + \sqrt{x^2+3x-3} = 1$

In this case, there are two radicals, isolate one, like:

$$\sqrt{x^2+5x-6} = 1 - \sqrt{x^2+3x-3}$$

Then square both sides:

$x^2 + 5x - 6 = 1 - 2\sqrt{x^2+3x-3} + x^2 + 3x - 3$
which simplifies to:
$2x - 4 = -2\sqrt{x^2+3x-3}$ or
$2 - x = \sqrt{x^2+3x-3}$ then square both sides!

$4 - 4x + x^2 = x^2 + 3x - 3$ and simplify!
$7 = 7x$ and $x = 1$. Substitute it and it does in fact work! Therefore, the solution is $x = 1$.

Solve each equation showing all work. <u>No Calculators!</u>

E1: $x = \sqrt{x+7} + 5$

E2: $4 = \sqrt[3]{x+2} + 8$

Section 2.3 – Solve each equation showing all work. <u>No Calculators!</u>

EOO

1. $\sqrt[3]{y-7} = 4$
2. $\sqrt{x+16} = \sqrt{x} + 4$
3. $\sqrt{9u-4} = \sqrt{7u-20}$
4. $\sqrt{4m^2-3m+2} - 2m - 5 = 0$
5. $\sqrt{a+21} - 1 = \sqrt{a+12}$
6. $2\sqrt[3]{7b-1} - 4 = 0$
7. $\sqrt{x+2} - 7 = \sqrt{x+9}$
8. $\sqrt{3x+10} = \sqrt{x+11} - 1$
9. $x + 5 = \sqrt{x+5} + 6$
10. $\sqrt{x-1} + x = 3$
11. $\sqrt{22-12x} + 5 = 2x$
12. $\sqrt{x} - \sqrt{7} = \sqrt{x+7}$
13. $\sqrt{x+3} + \sqrt{x-3} = 3$

14. $\sqrt{2x+5}+2\sqrt{x+6}=5$

15. $x^2=21-\sqrt{x^2-9}$

16. $\sqrt{x^2+3x+6}-\sqrt{x^2+3x-1}=1$

17. $\sqrt{x}+\sqrt{x-7}=\frac{21}{\sqrt{x-7}}$

18. $\frac{1}{1-x}+\frac{1}{1+\sqrt{x}}=\frac{1}{1-\sqrt{x}}$

19. $\frac{\sqrt{x}}{\sqrt{x}-1}+3=\frac{1}{\sqrt{x}-1}-1$

20. $\sqrt{2x+3}=5$

21. $\sqrt{3x-5}=4$

22. $\sqrt[3]{4x-1}=3$

23. $\sqrt[3]{x-2}=2$

24. $5\sqrt{x-1}=\sqrt{x+1}$

25. $\sqrt{x+14}-\sqrt{3x-10}=0$

26. $\sqrt{x-7}=\sqrt{x}-7$

27. $\sqrt{x+5}-1=\sqrt{x}$

28. $\sqrt{3x^2-4x+9}=3$

29. $\sqrt{x^2-9}=4$

30. $x+5=\sqrt{x+5}+6$

31. $x-2=\sqrt{x-2}+12$

32. $\sqrt{x-1}+3=x$

33. $\sqrt{-3x-14}-x=4$

34. $\sqrt{x-1}+x=3$

35. $\sqrt{x+6}-x=4$

36. $\sqrt{7-3x}+3=x$

37. $x-\sqrt{6-x}=4$

38. $\sqrt{3x-11}+3=x$

39. $\sqrt{3x+10}-4=x$

40. $\sqrt{22-12x}+5=2x$

41. $\sqrt{7-6x}+3x=2$

42. $\sqrt{x}+\sqrt{7}=\sqrt{x+7}$

43. $\sqrt{x+3}+\sqrt{x-3}=3$

44. $\sqrt{x+4}+\sqrt{x-4}=4$

45. $\sqrt{2x+25}-2\sqrt{x+4}=1$

46. $x^2=3-\sqrt{2x^2-3}$

47. $\sqrt{x^2-4x-12}-\sqrt{x^2-4x-5}=1$

48. $2\sqrt{x}-\sqrt{4x-3}=\frac{1}{\sqrt{4x-3}}$

49. $\frac{2}{\sqrt{x}+2}-\frac{\sqrt{x}}{2-\sqrt{x}}=\frac{x+4}{x-4}$

50. $\frac{1}{1-\sqrt{x}}=1-\frac{\sqrt{x}}{\sqrt{x}-1}$

51. $x=3+\sqrt{20-4x}$

52. $2\sqrt{2x}+1=\sqrt{4x+9}$

53. $\sqrt{2x}-\sqrt{x-3}=\frac{2}{\sqrt{x-3}}$

Section 2.4 – Solving Logarithmic and Exponential Equations

Example 1

Solve. $\log_8 48 - \log_8 w = \log_8 6$

Get the "variable log" on the side by itself:

$\log_8 48 - \log_8 6 = \log_8 w$ Use Quotient Property
$\log_8 48/6 = \log_8 w$ Simplify!
$\log_8 8 = \log_8 w$
Therefore w must be 8!

Example 2

Solve. $\log_{12} x = ½ \log_{12} 9 + 1/3 \log_{12} 27$

Use exponent rule first to simplify:
$\log_{12} x = \log_{12} 9^{1/2} + \log_{12} 27^{1/3}$ Simplify:
$\log_{12} x = \log_{12} 3 + \log_{12} 3$ Use Product Rule!
$\log_{12} x = \log_{12} 3*3$ or $\log_{12} 9$
Therefore, x must be 9.

Example 3

Solve. $\log_4 (x - 3) + \log_4 (x + 3) = 2$

First get the "logs" together into one log using the Product Rule:
$\log_4 (x-3)(x+3) = 2$ or
$\log_4 (x^2 - 9) = 2$ then convert to exponential form:
$4^2 = x^2 - 9$ or $25 = x^2$
and x = 5 or -5. If you put x = -5 back into the original equation we would be taking the log of a negative number which is NOT allowed, therefore our only solution is x = 5.

Solve each equation showing all work. No Calculators!

E1: $\log_p 64^{1/3} = ½$

E2: $\log_4 (2x + 11) = \log_4 (5x - 4)$

E3: $\log_{11} x + \log_{11} (x+1) = \log_{11} 6$

E4: $4e^{2x} = 5$

E5: $e^{2x} - 3e^x + 2 = 0$ (Hint: Let $e^x = y$ & substitute)

Section 2.4 – Solve each equation showing all work. No Calculators! Just get the equation to the point where you could enter it into a calculator.

1. $\log_x 49 = 2$
2. $\log_6 x + \log_6 9 = \log_6 54$
3. $\log_6 216 = x$
4. $\log_{10} \sqrt[3]{10} = x$
5. $\log_5 (x+4) + \log_5 8 = \log_5 64$
6. $\frac{1}{2}(\log_7 x + \log_7 8) = \log_7 16$
7. $5^x = 4^{x+3}$
8. $0.16^{4+3x} = 0.3^{8-x}$
9. $0.25 = \log 16^x$
10. $4^x = 16$
11. $7^x = \frac{1}{49}$
12. $\left(\frac{3}{4}\right)^x = \frac{27}{64}$
13. $\log_4 x = 3$
14. $\log_{10} x = -1$
15. $e^x = 10$
16. $2e^x = 39$
17. $e^x - 5 = 10$
18. $e^{3x} = 12$
19. $500e^{-x} = 300$
20. $e^{2x} - 4e^x - 5 = 0$
21. $3(1 + e^{2x}) = 4$
22. $\frac{400}{1+e^{-x}} = 200$
23. $10^x = 42$
24. $5^{-t/2} = 0.20$
25. $\frac{1}{3}(10^{2x}) = 12$
26. $3(5^{x-1}) = 21$
27. $\left(1 + \frac{0.10}{12}\right)^{12t} = 2$
28. $\ln x = 5$
29. $\ln 2x = 2.4$
30. $2 \log_6 4x = 0$
31. $\ln \sqrt{x+2} = 1$
32. $\ln x + \ln(x-2) = 1$
33. $\log_{10} (z-3) = 2$
34. $\log_{10} (x+4) - \log_{10} x = \log_{10} (x+2)$
35. $\log_3 x + \log_3 (x^2 - 8) = \log_3 8x$
36. $\ln(x+5) = \ln(x-1) - \ln(x+1)$
37. $\log_2 (x+5) - \log_2 (x-2) = 3$
38. $100(1 + 0.02)^{3+n} = 150$
39. $\ln (y+2) = \ln (y-7) + \ln 4$
40. $\ln (5+4y) - \ln (3+y) = \ln 3$
41. $2 \ln (x-3) = \ln (x+5) + \ln 4$
42. $\log_3 (a-3) = 1 + \log_3 (a+1)$
43. $\ln e^x - \ln e^3 = \ln e^5$
44. $\log_2 \sqrt{2y^2} - 1 = \frac{1}{2}$
45. $\log z = \sqrt{\log z}$

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46. $\log_3 3x = \log_3 36$
47. $\log_8 48 - \log_8 w = \log_8 6$
48. $\log_5 0.04 = x$
49. $\log_{12} x = \frac{1}{2} \log_{12} 9 + 1/3 \log_{12} 27$
50. $\log_4 (x-3) + \log_4 (x+3) = 2$
51. $2 \log_5 (x-2) = \log_5 36$
52. $2^x = 95$
53. $1/3 \log x = \log 8$
54. $4 \log (x+3) = 9$
55. $3^x = 243$
56. $8^x = 4$
57. $3^{x-1} = 27$
58. $\log_5 5x = 2$
59. $\ln (2x-1) = 0$
60. $e^x = 6500$
61. $4e^x = 91$
62. $e^x + 6 = 38$
63. $-14 + 3e^x = 11$
64. $e^{2x} = 50$
65. $1000e^{-4x} = 75$
66. $6e^{1-x} = 25$
67. $e^{2x} - 5e^x + 6 = 0$

EOO

EOO

68. $20(100 - e^{x/2}) = 500$

69. $\frac{3000}{2+e^{-2x}} = 1200$

70. $10^x = 570$

71. $6^{5x} = 3000$

72. $\ln x = -4.5$

73. $\ln 4x = 1$

74. $3 \log_2 5x = 10$

75. $\ln (x + 1)^2 = 2$

76. $\ln x + \ln (x+3) = 1$

77. $\log x^2 = 6$

78. $\log_2 x + \log_2 (x + 2) = \log_2 (x + 6)$

79. $\ln (x+1) - \ln (x-2) = \ln x^2$

80. $\log x - \log (2x - 1) = 0$

81. $\log_4 x - \log_4 (x-1) = ½$

Section 2.5: Solving Systems of Equations

Solve each system using any method that does NOT require a calculator! Remember Substitution Method or Linear Combination?

1. $y = x + 4$
   $3x + y = 16$

2. $x = y - 5$
   $3x + 2y = 3$

3. $4x + 3y = 31$
   $y = 2x + 7$

4. $x + 2y = 2$
   $5x - 3y = -29$

5. $6x - y = 31$
   $4x + 3y = 17$

6. $7x - 6y = -30$
   $x - 4y = -20$

7. $x + y = 23$
   $9x - 8y = 27$

8. $x - 3y = 13$
   $5x + 3y = 2$

9. $3(x - 2) + y = -4$
   $4x - 7y = 36$

10. $x + 6y = 19$
    $5(x - 7) + 2y = -24$

11. $2(x + 3) - y = 7$
    $7x - 3(y - 1) = 9$

12. $10x + 7y = -30$
    $8x + 7y = -24$

13. $5x - y = 22$
    $5x + 4y = -63$

EOO

14. $2.3x - 1.7y = 3.5$
    $4.7x - 1.7y = 10.7$

15. $10x - 4y = 35$
    $3x + 4y = 21$

16. $3x + 5y = 17$
    $2x + 3y = 11$

17. $4x - 5y = -19$
    $3x + 7y = 18$

18. $6a - 7b = 12$
    $5a - 4b = 10$

19. $2x + 9y = 12.5$
    $6x + 5y = 8.9$

20. $7u + 8v = 23$
    $3u - 2v = -1$

21. $4x - 3y = 2.7$
    $8x + 5y = 13.1$

22. $3x - 5y = -29$
    $2x - 10y = -42$

23. $4x + y = 42$
    $6x - 5y = 50$

24. $x + 12y = -8$
    $8x - 5y = 37$

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Answers by Section:

Section 1.1: 1. $18x^3$ 2. $10/x^7$ 3. $51/a^6$ 4. $-648x^2y^8$ 5. $648y^3/x^4$ 6. $b^8/3a^9$ 7. $5a^2-3a$ 8. $7x^5-4x$
9. xz^4/y^5 10. $x^4y/3z^2$ 11. 1 12. $12x^{1/6}$ 13. $30/x^{3/8}$ 14. $1728/x$ 15. $1/8x^{7/3}$ 16. $u^{6.6}p^3$ 17. $v^{3.5}/d^{0.6}$
18. $x^3y^{7/4}$ 19. $m^6k^{32/5}$ 20. 16 21. 32 22. 1/512 23. 16 24. -16 25. 1/64 26. 4 27. 1/729 28. -1/1000
29. 343/27 30. 16/81

Section 1.2: 1. $(2x+3)(x+1)$ 2. $(3a-1)(a-2)$ 3. $(7t-3)(t-1)$ 4. $(5x+1)(x-2)$ 5. $(2x-3)(x+1)$
6. $(2r+5)(3r+1)$ 7. $(6p+7)(p-1)$ 8. $(3u-1)(2u-3)$ 9. $(5n-12)(n-1)$ 10. $(5z+2)(2z+3)$
11. $(8x-3)(2x+3)$ 12. $2(2r-3)(r-1)$ 13. $2(2u+1)^2$ 14. $(3x-7y)(x-y)$ 15. $(2a+b)(a+5b)$
16. $(3r+2t)(r-t)$ 17. $(2w+3u)(w-5u)$ 18. $(4m-3n)^2$ 19. $2(7t-5s)(t+s)$ 20. $3(b-a)(3b+5a)$

Section 1.3: 1. $(x+2)/2$ 2. $(x+3)(x+5)/5x$ 3. $(x^3+6)/6x$ 4. -1 5. $-(x+7)/7$ 6. -1 7. $-(x+1)/(4+x)$
8. $(x-5y)^2/(xy(x+5y))$ 9. $(x+1)/(x-1)$ 10. $(x-1)/(x+4)$ 11. $x+2$ 12. $(x+5)/(5-x)$
13. $(x+5)^2/(x-7)^2$ 14. 1 15. $(x^2-y^2)^2$

Section 1.4: 1. $x/12$ 2. $2x/(x^2-1)$ 3. $9/(2x-3y)$ 4. $(2-4x)/(x^2-1)$ 5. 0 6. $y/(x-y)^2$ 7. $1/(x+4)$
8. $(3x+4y)/((x+y)(2x+3y))$ 9. 4 10. $-2x/(x-3)$ 11. $-x/a$ 12. $-1/(x^2-1)$ 13. $2/((x-1)(x-2)(x-3))$
14. $-2/((x-1)(x-3))$ 15. $3(x+3)/((x+2)(x-1))$ 16. $2x/(x^2-1)$ 17. $(x-1)/x$ 18. $x(x+1)/(x-2)$
19. $(x-1)/(x+1)$ 20. $6/x^2$

21. $1/6(a-b)$ 22. $1/(x+y)$ 23. $-4y/(x+y)^2$ 24. $-1/(1-4x^2)$ 25. $2b/(3a+2b)(3a-2)$ 26. $(7-3x)/(1-x)^2$
27. $x/(x-2y)^2$ 28. $1/(x-5)$ 29. $2xy/(x^3-8y^3)$ 30. $1/(x-1)$ 31. x 32. $4xy/(x-y)^2$ 33. $1/(x+2)$
34. $a/(3(a+b)(a-b))$ 35. $2/(x+y)$ 36. $1/(x+6)$ 37. $2/(x-2)(x-4)$ 38. $5(x+2)/(x+1)(x-4)$ 39. $14/(x^2-49)$
40. $1/x$ 41. $(2x-3)/(2x-5)$ 42. $(x+3)/(x-2)$ 43. $(x+2)/(x+1)$ 44. y^2-x^2/xy 45. y^2-x^2/x^2y^2
46. $2(x-3)/(x+2)$ 47. $2xy/x^2+y^2$

Section 1.5: 1. $(a+3)(a+b)$ 2. $(2x+3y)(4x+5z)$ 3. $(8x-3)(x-1)$ 4. $(x+3+y)(x+3-y)$
5. $(x-5+3a)(x-5-3a)$ 6. $(x+5)(3s+x-2)$ 7. $(2x+5)(3x+5)$ 8. Doesn't factor 9. $(2m-5)(m+10)$
10. $x^3(4x+3)(2x-1)$ 11. $3r(3u+2)^2$ 12. $2(x+3)(3x-2)(2x+1)$ 13. $(4x+15)(3x^2+8)$
14. $(2x-3)(9x^2+4)$ 15. $(7s-6)(r+1)(r-1)$ 16. $(2x-7)(5x+3)$ 17. $(8x+1)(x+6)$ 18. $(x+8)^2$
19. $(x+3)(x-3)$ 20. $36(x+2)(x-2)$ 21. $(6+(x-5))(6-(x-5))$ or $(x+1)(11-x)$
22. $(15x-2)(2x+3)$ 23. $(x-17)(x-4)$ 24. $(6x-1)(3x-2)$ 25. Doesn't factor
26. $4(x+2y)(x-2y)$ 27. $(10+x-y)(10-x+y)$ 28. $(3a+4y)(9a^2-12ay+16y^2)$

Section 1.5B: 1. $3(a+2)(a-1)$ 2. $(9x+2y)^2$ 3. $(c+2d)(c-2d)$ 4. $(2x+7y)(4x^2-14xy+49y^2)$
5. $(4x^2y-3z)(16x^4y^2+12x^2yz+9z^2)$ 6. $(z-3x-2y)(z+3x+2y)$ 7. $(4x+7y)(3x-8y)$
8. $(x^4+3y^4)(x^4-5y^4)$ 9. prime 10. $(x-a)(x+b)$ 11. $(a+b)(a^2-ab+b^2+a-b)$
12. $(5x-1+2y)(5x-1-2y)$ 13. $(a+2b+5c)(a+2b-5c)$ 14. $2b(a-4)^2$ 15. $2(2x-1)(2x+1)(2x^2+1)$
16. $(x^2+2)(x^2-2)(x^4+4)$ 17. $(a+b)(x+1)(x-1)$ 18. $-5(x^2+20)(x^2-20)$ 19. $(2x+2y-z)(3x+3y-z)$
20. $(y+2)(y-2)(y+3)(y-3)$ 21. $(5c+2)(3c-8)$ 22. prime 23. $(2x+1)(3x+1)(3x-1)$
24. $(5x-y^2)(25x^2+5xy^2+y^4)$ 25. $a(1+a)(1-a)$ 26. $(3x+8)(4x+1)$ 27. $2(x-2y)(x^2+2xy+4y^2)$
28. prime 29. $(x+y+1)(x-y+3)$ 30. $(x-1)(2x+1)(x+2)$

Section 1.6: 1. $\log_3 35$ 2. $\log_2 3$ 3. $\log_7 30$ 4. $\log_5 16$ 5. $\log_{12} 32$ 6. $\log_6 135$ 7. 8 8. -2 9. 9 10. 2001
11. $\ln 2x$ 12. $\log_4 \frac{z}{y}$ 13. $\log_2 (x+4)^2$ 14. $\ln \frac{x}{(x+1)^3}$ 15. $\ln \frac{x}{(x^2-4)^2}$ 16. $\ln \sqrt[3]{\frac{x(x+3)^2}{x^2-1}}$ 17. $\ln \frac{\sqrt[3]{y(y+4)^2}}{y-1}$
18. $\ln \frac{9}{\sqrt{x^2+1}}$ 19. 2 20. 2.4 21. 4.5 22. $\log_{10} 5 + \log_{10} x$ 23. $\log_{10} 5 - \log_{10} x$ 24. $4 \log_8 x$ 25. $\frac{1}{2} \ln z$
26. $\ln x + \ln y + \ln z$ 27. $\frac{1}{2} \ln(a-1)$ 28. $\ln z + 2 \ln(z-1)$ 29. $\frac{1}{3} \ln x - \frac{1}{3} \ln y$ 30. $4\ln x + \frac{1}{2} \ln y - 5 \ln z$
31. $\log_a \frac{xy}{m}$ 32. $\log_b \left(\frac{k}{ma}\right)$ 33. $\log_m \frac{a^2}{b^6}$ 34. $\log_y (p^{-7/6})$ 35. $\log_a [(z-1)^2(3z+2)]$ 36. $\log_b \left(\frac{2y+5}{\sqrt{y+3}}\right)$ 37. $\ln yz$
38. $\log_3 (5x)^{1/3}$ 39. $\ln \frac{x-2}{x+2}$ 40. $\log_5 \frac{8}{t}$ 41. $\ln [(x+1)^{1/2}(x-1)x^3]$ 42. $\log_6 (2x)^{-4}$ 43. $\log_7 (z-2)^{3/2}$
44. $\ln (5^{3/2} t^6)$ 45. 1/3 46. -3 47. ¾ 48. $2\log_b x - 2\log_b y - 3\log_b z$ 49. $1 + \log z$ 50. $\log y - \log 2$
51. $-3 \log_6 z$ 52. $1/3 \ln t$ 53. $\ln x + \ln y - \ln z$ 54. $\ln (x+1) + \ln (x-1) - 3\ln x$ 55. $\ln x - 3/2 \ln y$
56. $\ln x - \frac{1}{2} \ln (x^2 + 1)$ 57. $\ln x + \frac{1}{2} \ln(x + 2)$ 58. $\frac{1}{2} \log_b x + 4 \log_b y - 4 \log_b z$

Section 1.7: 1. $x^2-4x-2+-4/x-1$ 2. $3x^2 + x + 1$ 3. $-2x^3 + 6x^2 - 3x + 3 - \frac{7}{x+3}$ 4. $x^3 + 2x^2 + 5x + 9 + \frac{6}{x-2}$
5. $x^3 - x^2 + x - 1$ 6. $-x^5 - x^4 - x^3 - x^2 - x - 1 + \frac{3}{x-1}$ 7. $4x^2 - 2x - 2$ 8. $x^2 + 2x - 3$ 9. $x^2 + 3x$ 10. $x^2 + 9x - 1$

Section 1.8: 1. $2\sqrt{2}$ 2. $-\sqrt{3}/15$ 3. $3\sqrt{6}$ 4. $\sqrt{5}$ 5. $8\sqrt{3}$ 6. $6\sqrt{5}$ 7. $9+2\sqrt{14}$ 8. $8-2\sqrt{15}$
9. $29-12\sqrt{5}$ 10. $70+24\sqrt{6}$ 11. 2 12. 3 13. 10 14. 38 15. $42-6\sqrt{21}$ 16. $24-12\sqrt{2}$
17. $\frac{\sqrt{5}+1}{4}$ 18. $12+8\sqrt{2}$ 19. $\sqrt{7}-\sqrt{3}$ 20. $5\sqrt{3}+3\sqrt{2}$ 21. $\sqrt{6}+\sqrt{3}-\sqrt{2}-1$

22. $\frac{9\sqrt{6}+3\sqrt{3}-3\sqrt{2}-1}{17}$ 23. $\frac{107+42\sqrt{2}}{89}$ 24. $\frac{-13+7\sqrt{14}}{11}$ 25. $2+\sqrt{3}$ 26. $\frac{3-\sqrt{5}}{2}$

27. $2\sqrt{3}+3+\sqrt{21}$ 28. $\frac{2+\sqrt{2}+\sqrt{6}}{4}$ 29. $\frac{3\sqrt{2}+2\sqrt{3}+\sqrt{30}}{12}$ 30. $\frac{3\sqrt{5}+5\sqrt{3}-2\sqrt{30}}{30}$

31. $9\sqrt{5}$ 32. $5\sqrt{7}$ 33. $12\sqrt{6}$ 34. $7\sqrt{3}$ 35. $10\sqrt{5}$ 36. $8\sqrt{2}$ 37. $7\sqrt{3}$ 38. $-2\sqrt{7}$ 39. $\sqrt{5}$
40. $2\sqrt{2}-4\sqrt{3}$ 41. $\frac{3\sqrt{2}}{4}$ 42. $\frac{6\sqrt{5}}{5}$ 43. $\frac{4\sqrt{14}}{7}$ 44. $\frac{9\sqrt{2}}{10}$ 45. $3\sqrt{3}$ 46. $\sqrt{5}$ 47. $5\sqrt{6}$ 48. 0 49. $7\sqrt{7}$
50. $2\sqrt{2}$ 51. $12-15\sqrt{2}$ 52. $240-150\sqrt{3}$ 53. $52-14\sqrt{3}$ 54. $9+6\sqrt{2}$ 55. $113-30\sqrt{14}$ 56. -4
57. 11 58. 74 59. -57 60. -1000 61. $10\sqrt{6}-20$ 62. $4\sqrt{15}+12$ 63. $8+2\sqrt{3}$ 64. $-9\sqrt{2}+9\sqrt{7}$
65. $8\sqrt{11}+8\sqrt{5}$ 66. $\frac{2\sqrt{13}+7}{3}$ 67. $\frac{27-\sqrt{3}}{22}$ 68. $6+\sqrt{35}$ 69. $4-\sqrt{15}$ 70. $\frac{1421+4\sqrt{118326}}{355}$ *test your mult skills!!

Section 2.1: 1. 3/2, 1 2. 1/5, -2 3. 3/2, 1/3 4. -1/2 5. -2/3, ¾ 6. -9, 1 7. -1/2, -5/3 8. -2, 2 9. -4, 4
10. 3 11. -1/5, 2 12. -8, 11 13. ½, ¼ 14. $3/2 \pm \frac{\sqrt{37}}{2}$ 15. $\frac{5 \pm i\sqrt{11}}{2}$ 16. $\frac{1 \pm i\sqrt{35}}{4}$ 17. $\frac{-5 \pm \sqrt{97}}{4}$ 18. -4, 7
19. -1, 5/4 20. $\sqrt{6} \pm 2\sqrt{2}$ 21. -2/9, 3 22. 5/4, -5/4 23. 5/3, 2 24. -5, 3 25. 2 26. ½, -3 27. 7/5, -1
28. 2, -2 29. 1.5, -1/5 30. $\frac{5 \pm i\sqrt{83}}{6}$ 31. $-1 \pm i\sqrt{7}$ 32. ¼, -5 33. 6, -4 34. $\frac{6 \pm 2\sqrt{6}}{3}$ 35. 7, 3
36. 1/3, -3/2 37. 7/6 38. $\frac{-2 \pm 2\sqrt{7}}{3}$ 39. -5, -1 40. 1/6, -3/2 41. 0, -1/7 42. 0, -14/5 43. 1 44. -3
45. 4 46. 1, -3/2 47. 4, 2 48. 1, -5/3 49. 3/2, -1 50. 3

Section 2.2: 1. -34 2. 5/3 3. -1/2, 3 4. $\frac{-3\pm3\sqrt{2}}{2}$ 5. 5/13 6. -1 7. -1 8. no real soln 9. 7 10. 14/3
11. no real soln 12. all reals except 1 13. 3 14. 5 15. no real soln 16. -2 17. -43/6 18. -3 19. -4
20. all reals except 1 21. 23/15 22. -3 23. 0 24. -2,-1 25. 2 26. 6 27. no soln 28. 5,2 29. -2
30. 2 31. no soln 32. -1

Section 2.3: 1. 71 2. 0 3. no real soln 4. -1 5. 4 6. 9/7 7. no real soln 8. -2 9. 4 10. 2
11. no real soln 12. no real soln 13. 13/4 14. -2 15. $\pm3\sqrt{2}$ 16. 2, -5 17. 16 18. ¼ 19. no real soln
20. 11 21. 7 22. 7 23. 10 24. 13/12 25. 12 26. no solns 27. 4 28. 0, 4/3 29. 5,-5 30. 4 31. 18
32. 5 33. no soln 34. 2 35. -2 36. no solns 37. 5 38. 4, 5 39. -2, -3 40. no solns 41. -1/3 42. 0
43. 13/4 44. 5 45. 0 46. $\pm\sqrt{2}$ 47. *no soln* 48. 1 49. no solns 50. 0 51. $1+2\sqrt{3}$ 52. $3-\sqrt{5}$
53. $2+\sqrt{5}$

Section 2.4: 1. 7 2. 6 3. 3 4. 1/3 5. 4 6. 32 7. $\frac{3\log 4}{\log 1.25}$ 8. $\frac{\log_{0.16^4} 0.3^8}{\log(.16^3\text{-}0.3)}$ *or* $\frac{\log 0.100112915}{\log 0.0012288}$ 9. $\frac{1}{8\log 4}$ 10. 2

11. -2 12. 3 13. 64 14. 1/10 15. $\ln 10 \approx 2.303$ 16. $\ln 39/2 \approx 2.970$ 17. $\ln 15 \approx 2.708$ 18. $\frac{\ln 12}{3} \approx 0.828$

19. $\ln 5/3 \approx 0.511$ 20. $\ln 5 \approx 1.609$ 21. $\frac{1}{2}\ln 1/3 \approx -0.549$ 22. 0 23. $\log 42 \approx 1.623$ 24. 2

25. $\frac{1}{2}\log 36 \approx 0.778$ 26. $1+\frac{\log 7}{\log 5} \approx 2.209$ 27. $\frac{\log 2}{12\log\left(1+\frac{0.10}{12}\right)} \approx 6.960$ 28. $e^5 \approx 148.413$ 29. $\frac{e^{2.4}}{2} \approx 5.512$

30. ¼ = 0.250 31. $e^2 - 2 \approx 5.389$ 32. $1+\sqrt{1+e} \approx 2.928$ 33. 103 34. $\frac{-1+\sqrt{17}}{2} \approx 1.562$ 35. 4 36. no soln

37. 3 38. $\frac{\log_{1.02^3} 1.5}{\log 1.02}$ *or* $\frac{\log 1.413483502}{\log 1.02}$ 39. 10 40. 4 41. 11 42. no soln 43. 8 44. ±2 45. 1, 10
46. 12 47. 8 48. -2 49. 9 50. 5 51. 8 52. $\frac{\log 95}{\log 2} \approx 3.17$ 53. 8^3 or 512 54. $100\sqrt[4]{10} - 3$
55. $\frac{\log 243}{\log 3} = 5$ 56. 2/3 57. 4 58. 5 59. 1 60. ln 6500 61. ln (91/4) 62. ln 32 63. ln (25/3)
64. $\frac{\ln 50}{2} \approx 1.96$ 65. $\frac{\ln\left(\frac{75}{1000}\right)}{-4} \approx 0.65$ 66. 1-ln(25/6) 67. ln 2 or ln 3 68. 2 ln 75 69. $\frac{\ln 0.5}{-2} \approx 0.35$

70. log 570 71. $\frac{\log 3000}{5\log 6} \approx 0.89$ 72. $e^{-4.5}$ 73. e/4 74. $\frac{8\sqrt[3]{2}}{5}$ 75. e − 1 76. $\frac{-3\pm\sqrt{9+4e}}{2} \approx 0.73, -3.73$

77. 1000 78. 2 79. Stop at $x^3-2x^2-x-1=0$ Solve by Graphing (≈ 2.55) 80. 1 81. 2

Section 2.5: 1. (3,7) 2. (-1.4,3.6) or $\left(\frac{-7}{5},\frac{18}{5}\right)$ 3. (1, 9) 4. (-4, 3) 5. (5, -1) 6. (0, 5) 7. $\left(\frac{211}{17},\frac{180}{17}\right)$
8. (2.5, -3.5) or $\left(\frac{5}{2},\frac{-21}{6}\right)$ 9. (2, -4) 10. (1,3) 11. (3,5) 12. (-3, 0) 13. (1, -17) 14. (3, 2)

15. $\left(\frac{56}{13},\frac{105}{52}\right)$ 16. (4, 1) 17. (-1, 3) 18. (2, 0) 19. (0.4, 1.3) 20. (1,2) 21. (1.2, 0.7)

22. (-4, 3.4 or $\frac{17}{5}$) 23. (10, 2) 24. (4, -1)