Summer Review Work
For Students Entering Algebra 1 Honors

The attached packet is being given as a review of math topics that you should be comfortable with. These will be necessary skills for success in Algebra 1 Honors class next year.

Please do not wait until the last day of vacation to get started! On the other hand, do NOT attempt to complete the packet during the first week of vacation. This packet is designed to maintain your current knowledge of math concepts so that the topics discussed in the fall will be fresh in your mind.

If you have difficulties with the packet, use the examples given or feel free to search the internet for help on certain topics. Also, it may be beneficial to work with others.

Have an enjoyable summer!
I look forward to working with you this fall.

Ms. Ames
1 Basic Arithmetic

4 Key Terms
addition     factor
subtraction  prime number
multiplication  composite number
division     common multiple
sum          lowest common multiple
difference  common factor
product      highest common factor
dividend     fraction bar
divisor      numerator
quotient     denominator
remainder    proper fraction
mixed operations    improper fraction
bracket      mixed fraction
integer/whole number  complex fraction
multiple

1.1 Four Basic Arithmetic Operations

<table>
<thead>
<tr>
<th>Basic operation</th>
<th>Example</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>$3 + 9 = 12$</td>
<td>sum</td>
</tr>
<tr>
<td>Subtraction</td>
<td>$13 - 5 = 8$</td>
<td>difference</td>
</tr>
<tr>
<td>Multiplication</td>
<td>$2 \times 7 = 14$</td>
<td>product</td>
</tr>
<tr>
<td>Division</td>
<td>$\begin{array}{c} \text{dividend} \ 29 \div 6 = 4 \cdots 5 \end{array}$</td>
<td>\begin{array}{c} \text{divisor} \ \text{quotient} \end{array}</td>
</tr>
</tbody>
</table>

1
(b) In performing mixed operations, we should follow the order of operations below:

(i) Perform multiplication (×) and division (÷) first, then addition (+) and subtraction (−).

\[ \text{e.g. (1) } 30 - 6 \times 3 = 30 - 18 = 12 \quad \text{and} \quad \text{e.g. (2) } 5 + 14 \div 7 = 5 + 2 = 7 \]

(ii) When there are only addition / subtraction (or only multiplication / division) in an expression, perform the operations from LEFT to RIGHT.

\[ \text{e.g. (1) } 34 - 15 + 5 = 19 + 5 = 24 \quad \text{and} \quad \text{e.g. (2) } 0.28 \div 4 \times 3 = 7 \times 3 = 21 \]

(iii) When there are brackets in an expression, perform the operations inside the brackets first.

\[ \text{e.g. } 24 \div (4 \times 2) - 2 = 24 \div 8 - 2 = 3 - 2 = 1 \]

Example 1 Calculate the following.

(a) \( 8 \times 2.5 - 51 \div 3 \)

(b) \( 35 \div (16 - 3 \times 2) + 1.5 \)

Solution

(a) \( 8 \times 2.5 - 51 \div 3 \)

\[ = 20 - 17 \]

\[ = 3 \]

(b) \( 35 \div (16 - 3 \times 2) + 1.5 \)

\[ = 35 \div (16 - 6) + 1.5 \]

\[ = 35 \div 10 + 1.5 \]

\[ = 3.5 + 1.5 \]

\[ = 5 \]
Example 7

In each flat of a building, there are a living room with area 50 m² and two bedrooms with area 10 m² each.

(a) Find the total area of the flat.
(b) If the building has 21 flats, find the sum of the areas of these flats.

Solution
(a) Total area = 10 + 10 + 50
    = 70 (m²)

(b) Sum of areas = 70 × 21
    = 1470 (m²)

Let's Try 1.1

Calculate the following. [Nos. 1–4]

1. 28 − 19 + 7
   =

2. 14 + 8 × 12 − 55
   = 14 + [ ] − 55

3. 16 − (24 − 5 × 3)
   = [ ] − ( [ ] − [ ] )

4. (8 − 5.4) ÷ 13 × 2
   =

5. Mr Wong orders 3 hot dogs and 2 cans of coke in a fast food shop. The coke is sold at $5 per can and Mr Wong pays $46 in total. Find the price of a hot dog.

   $ 7  $ 5.00
1.2 Multiples and Factors

(a) Multiples

\[ 6 \times 1 = 6 \]
\[ 6 \times 2 = 12 \quad \text{Multiples of 6} \]
\[ 6 \times 3 = 18 \]

\[ \vdots \]

\[ \therefore \text{The first 3 multiples of 6 are 6, 12 and 18.} \]

(b) Factors

(i) Consider the following expression.
\[ 8 \div 2 = 4 \quad \text{Remainder is 0.} \]

\[ 4 \text{ is an integer.} \]

\[ \therefore 8 \text{ is divisible by 2.} \]

(ii) Consider the following expression.
\[ 12 \div 4 = 4 \quad \text{\Delta 12 is divisible by 3.} \]

\[ \therefore 3 \text{ is a factor of 12.} \]

\[ \text{e.g.} \quad 12 = 1 \times 12 \]
\[ = 2 \times 6 \]
\[ = 3 \times 4 \]

\[ \therefore \text{Factors of 12 are 1, 2, 3, 4, 6 and 12.} \]

(c) Prime Numbers and Composite Numbers

(i) Numbers having only two factors (1 and itself) are called prime numbers.
\[ \text{e.g.} \quad \text{Prime numbers up to 20 are 2, 3, 5, 7, 11, 13, 17 and 19.} \]

(ii) Numbers having 3 or more factors (including 1) are called composite numbers.
\[ \text{e.g.} \quad \text{Composite numbers up to 10 are 4, 6, 8, 9 and 10.} \]

<table>
<thead>
<tr>
<th>Number</th>
<th>Factor</th>
<th>Prime Numbers</th>
<th>Composite Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>2</td>
<td>1, 2</td>
<td>✔️</td>
<td>X</td>
</tr>
<tr>
<td>3</td>
<td>1, 3</td>
<td>✔️</td>
<td>X</td>
</tr>
<tr>
<td>4</td>
<td>1, 2, 4</td>
<td>X</td>
<td>✔️</td>
</tr>
<tr>
<td>5</td>
<td>1, 5</td>
<td>✔️</td>
<td>X</td>
</tr>
<tr>
<td>6</td>
<td>1, 2, 3, 6</td>
<td>X</td>
<td>✔️</td>
</tr>
</tbody>
</table>
(d) **Lowest Common Multiple (L.C.M.)**

Multiples of 6 are 6, 12, (18), 24, 30, (36), ...
Multiples of 9 are 9, (18), 27, (36), 45, ...

The circled numbers 18 and 36 are called the common multiples of 6 and 9.
The smallest common multiple is called the lowest common multiple (abbreviated as L.C.M.).

\[ \therefore \text{ The L.C.M. of 6 and 9 is 18.} \]

(e) **Highest Common Factor (H.C.F.)**

Factors of 18 are (1), (2), (3), (6), 9, 18
Factors of 24 are (1), (2), (3), 4, (6), 8, 12, 24

The circled numbers 1, 2, 3 and 6 are called the common factors of 18 and 24.
The largest common factor is called the highest common factor (abbreviated as H.C.F.).

\[ \therefore \text{ The H.C.F. of 18 and 24 is 6.} \]

**Example (3)** Write down the factors of 12 and 32. Hence, find the H.C.F. of 12 and 32.

**Solution**

Factors of 12 are 1, 2, 3, 4, 6 and 12.
Factors of 32 are 1, 2, 4, 8, 16 and 32.

\[ \therefore \text{ The H.C.F. of 12 and 32 is 4.} \]

**Example (4)** Write down the first 5 multiples of 16 and 20. Hence, find the L.C.M. of 16 and 20.

**Solution**

The first 5 multiples of 16 are 16, 32, 48, 64 and 80.
The first 5 multiples of 20 are 20, 40, 60, 80 and 100.

\[ \therefore \text{ The L.C.M. of 16 and 20 is 80.} \]
1.2 Let's Try

1. Write down the factors of 14 and 35. Hence, find the H.C.F. of 14 and 35.
   **Solution**
   Factors of 14 are
   Factors of 35 are
   \[ \therefore \] The H.C.F. of 14 and 35 is

2. Write down the first 5 multiples of 8 and 10. Hence, find the L.C.M. of 8 and 10.
   **Solution**
   The first 5 multiples of 8 are
   The first 5 multiples of 10 are
   \[ \therefore \] The L.C.M. of 8 and 10 is

3. Write down all the prime numbers from 20 to 30.
   **Solution**
   The prime numbers from 20 to 30 are

1.3 Fractions

(a) Types of Fractions

The following are 3 common types of fractions:

<table>
<thead>
<tr>
<th>Type</th>
<th>Meaning</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proper fraction</td>
<td>a fraction with a numerator less than the denominator</td>
<td>( \frac{1}{4}, \frac{2}{7}, \frac{8}{15} )</td>
</tr>
<tr>
<td>Improper fraction</td>
<td>a fraction with a numerator greater than or equal to the denominator</td>
<td>( \frac{7}{7}, \frac{11}{6}, \frac{10}{4} )</td>
</tr>
<tr>
<td>Mixed fraction</td>
<td>a sum of a natural number and a proper fraction</td>
<td>( 1\frac{2}{7}, 3\frac{1}{5}, 12\frac{3}{8} )</td>
</tr>
</tbody>
</table>

\[ \text{Note}: \] In a fraction, if the numerator, denominator or both contain a fraction, the fraction is called a complex fraction.

\[ \frac{3}{9} \frac{5}{10} \] is a complex fraction and \[ \frac{3}{9} \frac{5}{10} = \frac{3}{5} \div \frac{9}{10} \]

\[ \quad \]
(b) Operations with Fractions

(i) Addition or subtraction:
Expand the fractions to make their denominators the same first, then add or subtract the numerators.

**Example 5** Calculate \( \frac{1}{2} - \frac{2}{7} \).

\[
\frac{1}{2} - \frac{2}{7} = \frac{7 - 4}{14} = \frac{3}{14}
\]

The L.C.M. of 2 and 7 is 14.

\[
\therefore \frac{1}{2} = \frac{1 \times 7}{2 \times 7} = \frac{7}{14} \quad \frac{2}{7} = \frac{2 \times 2}{7 \times 2} = \frac{4}{14}
\]

(ii) Multiplication or division:
Convert all mixed fractions into improper fractions first, then cancel out all the common factors in the numerators and the denominators.

**Example 6** Calculate the following.

(a) \( \frac{2}{3} \)

(b) \( \frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \div 1\frac{1}{3} \)

**Solution (a)**

\[
\frac{2}{3} \times \frac{4}{9} = \frac{2 \times 4}{3 \times 9} = \frac{8}{27}
\]

To divide a fraction by another, turn the divisor upside down and convert \( : \) into \( \times \).
(b) \[
\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \div 1 \frac{1}{3}
\]

Convert into improper fraction first.

\[
= \frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{4}{3}
\]

\[
= \frac{1}{8} + \frac{5}{8}
\]

\[
= \frac{6}{8}
\]

Simplify the answer.

\[
= \frac{3}{4}
\]

Let's Try 1.3

Calculate the following.

1. \[
\frac{4}{9} + \frac{5}{12}
\]

2. \[
2 \frac{4}{7} - 1 \frac{1}{3}
\]

3. \[
2 \frac{5}{8} \times 1 \frac{2}{7}
\]

4. \[
\frac{2}{3} \div \frac{14}{45}
\]

5. \[
1 - 1 \frac{1}{5} \div 12
\]

6. \[
\frac{7}{9} \times 3 + 3 \frac{1}{2} \div 14
\]
Exercise 1

Calculate the following. [Nos. 1–8]

1. \[ 15 \times 5 \div 3 \]

2. \[ 2.5 - 1.4 + 3 \times 0.3 \]

3. \[ 50 - 9.2 \times 5 + 13 \]

4. \[ (5.2 + 4.3) \div 5 \]

5. \[ \frac{3}{7} \]

6. \[ \left( 2\frac{1}{2} - 3 \times \frac{2}{3} \right) \div \frac{1}{2} \]

7. \[ \left( 8\frac{2}{5} - 4\frac{1}{5} \times \frac{11}{7} \right) \div \frac{7}{12} \]

8. \[ 3\frac{1}{2} \times \left( 2\frac{1}{2} + \frac{5}{6} \right) \div \left( \frac{1}{6} \times \frac{1}{3} \right) \]

Solution Factors of 15 are ____________________________
Factors of 30 are ____________________________

∴ The H.C.F. of 15 and 30 is ____________________________


Solution The first 8 multiples of 12 are ____________________________
The first 8 multiples of 28 are ____________________________
∴ The L.C.M. of 12 and 28 is ____________________________

11. Write down all the composite numbers from 31 to 59.

Solution Composite numbers from 31 to 59 are ____________________________

Fill in the □ with ‘+’, ‘−’, ‘×’ or ‘÷’ to make the both sides of the following expressions equal. [Nos. 12–13]

12. \[ \frac{1}{11} \Box 8 \Box \frac{10}{11} \times \frac{1}{8} = \frac{1}{8} \]

13. \[ \left( \frac{1}{2} \Box \frac{1}{3} + \frac{1}{6} \right) \Box 36 = 12 \]

14. Taxi Fare Table

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>First 2 km</td>
<td>$22.00</td>
</tr>
<tr>
<td>Every subsequent 0.2 km</td>
<td>$1.60</td>
</tr>
<tr>
<td>Every piece of baggage</td>
<td>$5.00</td>
</tr>
</tbody>
</table>

City A and city B are 4 km apart. City B and city C are 13.2 km apart. Paco took a taxi from A to C via B without any baggage. How much taxi fare should he pay?
Basic Skills Practice
Decimals and Percents, Part 1

You change a decimal to a percent by multiplying the decimal by 100 and adding a percent sign.

Example 1: Multiplying by 100 moves the decimal point two places to the right.
            \[0.34 \times 100 = 34, \text{ so } 0.34 = 34\%\]

You change a percent to a decimal by dividing the percent by 100 and omitting the percent sign.

Example 2: Dividing by 100 moves the decimal point two places to the left.
            \[25 \div 100 = 0.25, \text{ so } 25\% = 0.25\]

Write each decimal as a percent.

1. 0.05
2. 0.22
3. 0.75
4. 0.17
5. 0.41
6. 0.03

Write each percent as a decimal.

7. 82%
8. 37%
9. 7%
10. 59%
11. 3%
12. 68%

Write a percent for each decimal.

13. 0.544
14. 0.0065
15. 0.085
16. 0.105
17. 0.014
18. 1.75

Write a decimal for each percent.

19. 3.4%
20. 17.8%
21. 100%
22. 83%
23. 0.16%
24. 755.2%

Solve each problem.

25. What is 5.7% written as a decimal?  
26. What is the correct way to write 0.75 as a percent?
A percent is a ratio that compares a number with 100. To write a percent as a fraction, write the number as the numerator and 100 as the denominator.

41\% = \frac{41}{100}

7\% = \frac{7}{100}

After you write the fraction, reduce it to lowest terms.

25\% = \frac{25}{100} = \frac{1}{4}

Percents may be greater than 100. Write as an improper fraction, and then reduce.

110\% = \frac{110}{100} = \frac{11}{10} = 1\frac{1}{10}

Write each percent as a fraction.

1. 13\%  
2. 23\%  
3. 37\%  
4. 11\%  
5. 3\%  
6. 99\%  
7. 49\%  
8. 7\%  
9. 39\%

Write each percent as a fraction. Then reduce to lowest terms.

10. 50\%  
11. 75\%  
12. 66\%  
13. 12\%  
14. 24\%  
15. 40\%

Write each percent as a fraction or mixed number in reduced form.

16. 160\%  
17. 175\%  
18. 125\%  
19. 100\%  
20. 113\%  
21. 500\%

Solve each problem.

22. Kay has served as mayor for 75\% of her term. What fraction of her term has she served?

23. Mr. Renwiz phoned 27 students to tell them that the school was closed due to flooding. Each call took approximately 10\% of an hour. What fraction of an hour was Mr. Renwiz on the phone for each call?
Basic Skills Practice

Using Ratios and Rates, Part 1

A ratio is a comparison of two numbers or quantities. Ratios may be written three different ways:

\[ \frac{2}{7} \quad 2 \text{ to } 7 \quad 2 : 7 \]

Ratios can be used to make predictions.

**Example 1:** Arthur makes 2 goals for every 7 attempts. Predict how many goals will he make in 35 attempts.

\[
\text{Compare: } \frac{\text{goals}}{\text{attempts}} = \frac{2}{7}
\]

Make equivalent ratios. (Making equivalent ratios is like making equivalent fractions.)

<table>
<thead>
<tr>
<th>Goals</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attempts</td>
<td>7</td>
<td>14</td>
<td>21</td>
<td>28</td>
<td>35</td>
</tr>
</tbody>
</table>

We can expect Arthur to make 10 goals in 35 attempts.

Ratios can be used to determine how many times greater one number is than another.

**Example 2:** Richard is 16 years old. Martha is 4 years old. How many times older is Richard than Martha?

\[
\text{Compare: } \frac{\text{Richard's age}}{\text{Martha's age}} = \frac{16}{4} \quad \text{Divide: } \frac{16}{4} = 4
\]

Richard is 4 times older than Martha.

Ratios can be used to compare amounts when the units of measure are the same.

**Example 3:** A punch recipe uses 4 cups of water and 16 ounces of orange concentrate.

What is the ratio of water to concentrate?

Make sure the units are the same. Change 16 ounces to 2 cups.

\[
\text{Compare: } \frac{\text{water}}{\text{concentrate}} = \frac{4 \text{ cups}}{2 \text{ cups}} = \frac{2 \text{ cups}}{1 \text{ cup}}
\]

The ratio of water to concentrate is 2 cups of water to 1 cup of concentrate.

A ratio that compares two different kinds of quantities is called a rate.

Examples of rates: \( \frac{\text{miles}}{\text{hour}} \), \( \frac{\text{miles}}{\text{gallon}} \)

When the rate has a denominator of 1, it is called a unit rate.

**Example 4:** A car travels 300 miles on 12 gallons of gasoline. What is the unit rate in miles per gallon (mpg)?

A unit rate compares a quantity to a unit of one.

\[
\text{Compare: } \frac{\text{miles}}{\text{gallons}} = \frac{300}{12} = 25 \text{ mpg}
\]

Write three equivalent ratios for each given ratio.

1. \( \frac{5}{6} \)  
2. \( \frac{2}{3} \)

Write a ratio that compares each quantity.

3. the number of vowels to the number of consonants in the alphabet
4. the number of months that end in y to the total number of months in a year
Basic Skills Practice

Using Proportions, Part 1

A proportion is an equation stating that two ratios are equal.

\[ \frac{1}{2} \text{ and } \frac{4}{8} \text{ form the proportion } \frac{1}{2} = \frac{4}{8} \]

Cross products can be used to tell if two ratios form a proportion.

\[ \frac{1}{2} = \frac{4}{8} \quad 2 \times 4 = 8 \quad 1 \times 8 = 8 \quad \text{Yes} \]

\[ \frac{1}{3} = \frac{4}{5} \quad 3 \times 4 = 12 \quad 1 \times 5 = 5 \quad \text{No} \]

Cross products can be used to find the missing term in a proportion.

\[ \frac{18}{n} = \frac{6}{3} \]

Example 1: Find the value of \( n \) in \( \frac{18}{n} = \frac{6}{3} \).

Use cross products: \( 6 \times n = 18 \times 3 \)

\[ 6n = 54 \]

\[ n = 9 \]

Proportions can help solve problems.

Example 2: What is the cost of a dozen oranges if 3 oranges cost 99 cents?

\[ \frac{3}{99} = \frac{12}{n} \]

\[ 3n = 1188 \]

\[ n = 396 \]

12 oranges cost 396 cents or $3.96

---

Does each pair of ratios form a proportion? Write yes or no.

1. \( \frac{3}{9} \quad \frac{6}{18} \)

2. \( \frac{9}{10} \quad \frac{18}{30} \)

3. \( \frac{1}{2} \quad \frac{50}{100} \)

4. \( \frac{10}{30} \quad \frac{20}{40} \)

5. \( \frac{55}{121} \quad \frac{5}{11} \)

6. \( \frac{0.4}{2.3} \quad \frac{1.6}{9.2} \)

---

Solve each equation for \( n \).

7. \( \frac{48}{n} = \frac{4}{7} \)

8. \( \frac{9}{24} = \frac{n}{48} \)

9. \( \frac{4}{18} = \frac{6}{n} \)

10. \( \frac{n}{55} = \frac{18}{22} \)

11. \( \frac{5.1}{n} = \frac{1.7}{2} \)

12. \( \frac{16}{34} = \frac{n}{1.7} \)

---

Solve by using a proportion.

13. Joe's favorite flavor of frozen yogurt is chocolate fudge. There are 65 calories in 2 ounces of chocolate fudge frozen yogurt. How many calories are there in 10 ounces?

14. One roll of gift wrap will wrap 4 shirt boxes. How many rolls will be needed to wrap 24 shirt boxes?

15. Elissa drives 164 miles in 4 hours. At that rate, how many miles will she travel in 6.5 hours?
Basic Skills Practice

Similar Figures, Part 1

Two or more figures that have the same shape but different sizes are called similar figures. Triangle $ABC$ and triangle $DEF$ are similar figures. This is written as $\triangle ABC \sim \triangle DEF$.

When two figures are similar,
- the corresponding angles are congruent.
  $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$
- the ratios of the lengths of the corresponding sides are equal.
  \[
  \frac{AB}{DE} = \frac{5}{10} = \frac{1}{2}, \quad \frac{BC}{EF} = \frac{4}{8} = \frac{1}{2}, \quad \frac{AC}{DF} = \frac{3}{6} = \frac{1}{2}
  \]

You can determine if two figures are similar by checking to see that they meet both conditions:

**Example 1:** Are rectangle $ABCD$ and rectangle $EFGH$ similar?

- Because both figures are rectangles, all angles are right angles, so they are all congruent.
- $\frac{AB}{EF} = \frac{2}{3}, \quad \frac{BC}{FG} = \frac{4}{6} = \frac{2}{3}, \quad \frac{CD}{GH} = \frac{2}{3}, \quad \frac{DA}{HE} = \frac{4}{6} = \frac{2}{3}$

The ratios of the corresponding sides are equal, so rectangle $ABCD \sim$ rectangle $EFGH$.

You can find the length of the missing side in similar figures by using proportions.

**Example 2:** The two flags are similar. What is the length of $s$?

Write a proportion. \( \rightarrow \frac{2}{3} = \frac{4}{s} \)
Solve the proportion. \( \rightarrow 2s = 12 \)
\( s = 6 \)

Determine if the pairs of figures are similar. Write yes or no in the blank spaces.

1. 
   \[ \frac{3}{4} \quad \frac{4.5}{6} \]

2. 
   \[ \frac{2}{3} \quad \frac{4}{5} \]

Each pair of figures is similar. Find the length of the missing side.

3. 
   \[ \frac{8}{12} \quad \frac{6}{x} \]

4. 
   \[ \frac{12}{16} \quad \frac{4}{x} \]
Problem Solving

Indirect Measurement

Answer each question. Draw diagrams for all these problems.

1. On a sunny day, a tower casts a shadow 4,428 feet long. A 6-foot-tall person standing nearby casts an 18-foot-long shadow. How tall is the tower?

Solution:

\[
\frac{6}{18} = \frac{x}{4,428}
\]

\[
6 \cdot 4,428 = 18x
\]

\[
x = \frac{4,428}{3}
\]

1,476 ft

3. The world's tallest man cast a shadow that was 535 inches long. A woman stood next to him. She was 5 feet 4 inches tall and cast a shadow that was 320 inches long. How tall was the world's tallest man in feet and inches?

2. On a sunny day a building casts a shadow that is 2,908 feet long. A 5-foot-tall person standing by the building casts a 10-foot-long shadow. How tall is the building?

Drawing diagrams will help me solve these problems.

4. Hoover Dam on the Colorado River casts a shadow that is 2,904 feet long. An 18-foot-tall flagpole next to the dam casts a shadow that is 72 feet long. How tall is Hoover Dam?

Circle the letter of the correct answer.

5. A 6-foot-tall man casts a shadow that is 30 feet long. A boy standing next to the man casts a shadow that is 12 feet long. How tall is the boy?

A 2.2 feet

B 5 feet

C 2.4 feet

6. An ostrich is 108 inches tall. Its shadow is 162 inches. An emu standing next to it casts a 90-inch shadow. How tall is the emu?

A 162 inches

B 90 inches

C 60 inches
1. Use similar triangles to find the height of the lamp post.

\[ \frac{h}{20} = \frac{5}{10} \]

\[ 10h = 100 \]

\[ h = 10 \text{ feet} \]

2. Use similar triangles to find the height of the man.

3. A 3-foot-tall boy looks into a trick mirror that makes a person appear shorter. The boy appears to be 1 foot tall in the mirror. If a man appears to be 2 feet tall in the mirror, what is his actual height?

4. A carnation casts a shadow that is 20 inches long. At the same time, a 3-inch-tall tulip casts a shadow that is 12 inches long. How tall is the carnation?

5. A bicycle casts a shadow that is 8 feet long. At the same time, a girl who is 5 feet tall casts a shadow that is 10 feet long. How tall is the bicycle?

6. A sand castle casts a shadow that is 5 inches long. A 15-inch-tall bucket sitting next to the sand castle casts a shadow that is 3 inches long. How tall is the sand castle?
Key Terms

length
width/breadth
base
height
upper base
lower base
splitting method
filling method
capacity
container
dimensions
depth
maximum

6.1 Perimeters of Simple Plane Figures

(a) Perimeter of a plane figure = sum of the lengths of all its sides

<table>
<thead>
<tr>
<th>Plane figure</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>length</td>
<td>Perimeter of a square = length × 4</td>
</tr>
<tr>
<td>width</td>
<td>Perimeter of a rectangle = (length + width) × 2</td>
</tr>
</tbody>
</table>

(b) mm, cm, m and km are common measuring units of lengths.

Example 1 The figure shows a rectangle formed by 3 squares with side 8 cm. Find the perimeter of the rectangle.

Solution

Length of the rectangle = 8 × 3
= 24 (cm)

Perimeter of the rectangle = (24 + 8) × 2
= 64 (cm)
Example 2] Andy uses a wire of 48 mm long to form a square. Find the length of the square.

Solution Let \( x \) mm be the length of the square.

\[
\begin{align*}
4x &= 48 \\
\frac{4x}{4} &= \frac{48}{4} \\
x &= 12
\end{align*}
\]

\( \therefore \) The length of the square is 12 mm.

Let's Try 6.1

Find the perimeters of the following figures. [Nos. 1–4]

1. \[ \begin{array}{c}
6 \text{ cm} \\
\end{array} \]

2. \[ \begin{array}{c}
2.8 \text{ cm} \\
2.6 \text{ cm} \\
4.6 \text{ cm} \\
\end{array} \]

3. \[ \begin{array}{c}
6 \text{ cm} \\
10 \text{ cm} \\
8 \text{ cm} \\
\end{array} \]

4. \[ \begin{array}{c}
4 \text{ mm} \\
4 \text{ mm} \\
2.4 \text{ mm} \\
3 \text{ mm} \\
\end{array} \]

5. The perimeter of a rectangle is 110 cm. If the length is 42 cm, find the width.

Solution Let \( y \) cm be the width.

\[
\begin{array}{c}
\square + \square \times \square = \square \\
\end{array}
\]

\( \therefore \) The width is \( \text{ cm} \).
### 6.2 Areas of Simple Plane Figures

<table>
<thead>
<tr>
<th>Plane figure</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Square" /></td>
<td>Area of a square = length × length</td>
</tr>
<tr>
<td><img src="image" alt="Rectangle" /></td>
<td>Area of a rectangle = length × width</td>
</tr>
<tr>
<td><img src="image" alt="Parallelogram" /></td>
<td>Area of a parallelogram = base × height</td>
</tr>
<tr>
<td><img src="image" alt="Triangle" /></td>
<td>Area of a triangle = ( \frac{1}{2} \times \text{base} \times \text{height} )</td>
</tr>
<tr>
<td><img src="image" alt="Trapezium" /></td>
<td>Area of a trapezium = ( \frac{1}{2} \times (\text{upper base} + \text{lower base}) \times \text{height} )</td>
</tr>
</tbody>
</table>

(b) mm², cm², m² and km² are common measuring units of areas.

**Example 5**

The polygon on the right is formed by a parallelogram and a triangle. Find the area of the polygon.

**Solution**

Area of the parallelogram = \( 12 \times 4 \) = 48 (m²)

Area of the triangle

\[
= \frac{1}{2} \times (12 - 6) \times (7 - 4)
\]

\[
= \frac{1}{2} \times 6 \times 3
\]

\[
= 9 \text{ (m²)}
\]

\[
\therefore \text{ Area of the polygon } = 48 + 9 = 57 \text{ (m²)}
\]
Example 4: In the figure, John cuts out a trapezium from a piece of rectangular paper. What is the area of the remaining part?

Solution:
Area of the rectangle $= 20 \times 18$
$= 360 \text{ (cm}^2\text{)}$

Area of the trapezium $= \frac{1}{2} \times (10 + 16) \times (18 - 5)$
$= \frac{1}{2} \times 26 \times 13$
$= 169 \text{ (cm}^2\text{)}$

Area of the remaining part $= 360 - 169$
$= 191 \text{ (cm}^2\text{)}$

Let's Try 6.2
Find the areas of the following figures.

1. 

2. 

3. 

4. 

5. 

6. 

- 42 -
6.3 Volumes of Simple Solid Figures

(a) | **Solid figure** | **Volume** |
---|---|---|
| ![Cube Diagram] | Volume of a cube = length × length × length |
| ![Cuboid Diagram] | Volume of a cuboid = length × width × height |

(b) mm³, cm³, m³ and km³ are common measuring units of volumes. For the capacity of a container or volume of liquid, the units mL and L can also be used.

Example 5 The solid below is formed by 6 cuboids of the same size. Find the volume of the solid.

```
Example 5: The solid below is formed by 6 cuboids of the same size. Find the volume of the solid.
```

```
SOLUTION

Length of each cuboid = 12 ÷ 3  
= 4 (m)  
Volume of each cuboid = 4 × 3 × 2  
= 24 (m³)  
∴ Volume of the solid = 24 × 6  
= 144 (m³)
```

-43-
Example 6. The dimensions of the container below are 50 cm × 40 cm × 25 cm. If Dick pours 30 L of water into the container, find the depth of water in the container.

Solution. Let $d$ cm be the depth of water in the container.

\[
50 \times 40 \times d = 30 \times 1 \, 000 \quad \Rightarrow 30 \text{ L} = 30 \times 1 \, 000 \text{ cm}^3
\]

\[
2 \, 000d = 30 \, 000
\]

\[
\frac{2 \, 000d}{2 \, 000} = \frac{30 \, 000}{2 \, 000}
\]

\[
d = 15
\]

\[
\therefore \text{ The depth of water in the container is 15 cm.}
\]

Let's Try 6.3

Find the volumes of the following solids. [Nos. 1-2]

1. 

2. 

3. (a) In the figure, the capacity of the container is _______ L.

(b) Pansy pours a bottle of 1.5 L orange juice into the container. The depth of orange juice in the container is _______ cm.
Exercise 6

Find the perimeters of the following figures. [Nos. 1–2]

1. 

2. 

3. In the figure, the length of each small square is 1 cm.
   (a) Perimeter of the shaded region is \( \__ \) cm.
   (b) Area of the shaded region is \( \__ \) cm\(^2\).

In the following figures, the length of each small square is 1 cm. Find the areas of the shaded regions. [Nos. 4–5]

4. 

5. 

6. The volume of the gift box on the right is \( \__ \) cm\(^3\).

7. The figure shows the remaining part of a cube after a cuboid is cut out. The volume of this remaining part is \( \__ \) cm\(^3\).
8. David gives a box of candies to Mandy. The box is a cuboid with length 10 cm, width 6 cm and volume 180 cm$^3$. Find the height of the box.

9. A fence of 200 m is built around a rectangular garden. If the length of the garden is 60 m, find the area of the garden.

10. Jack uses a piece of wire to form an equilateral triangle of side 14 cm. Then, Michelle reforms it to a square. What is the area of the square?

11. The figure shows a rectangle with length 12 cm and width 8 cm. It is cut into one square and two trapeziums of different sizes. If the side of the square is 4 cm, what is the area of the smaller trapezium?

12. There is a box with length 20 cm, width 12 cm and height 10 cm. Find the maximum number of blocks as shown on the right can be put into the box.
名称

**Practice**

**Exponents**

Name the base and the exponent for each of the following. The first one is done for you.

1. \(7^2\)  
2. \(5^4\)  
3. \(6^8\)

- **base**: 7  
- **exponent**: 2

Write using exponents. The first one is done for you.

4. \(4 \times 4\)  

5. \(2 \times 2 \times 2\)

6. \(10 \times 10\)

- **Answer**: \(4^2\)

7. \(5 \times 5 \times 5 \times 5\)

8. \(3 \times 3 \times 3 \times 3\)

9. \(8 \times 8 \times 8 \times 8 \times 8\)

Write as repeated multiplication. The first one is done for you.

10. \(6^2\)

11. \(5^3\)

12. \(2^8\)

- **Answer**: \(6 \times 6\)

13. How many different ways can you use the digits 3 and 5 to write expressions in exponential form? What are the expressions?
Problem Solving

Exponents

1. The surface temperature of the Sun is close to 10,000°F. Write 10,000 using exponents.

Solution:
Move the decimal point to the left so the number is between 0 and 10.
10,000 → 1.0000
The decimal point moved 4 places. Use 4 as the exponent.
1.0 × 10^4

2. Patty Berg has won 4^2 major women’s titles in golf. Write 4^2 in standard form.
The exponent 2 means to write the base ______ as a factor 2 times:
_______ × _______ = _______

3. William has 3^3 baseball cards and 4^3 football cards. Write the number of baseball cards and football cards that William has.

4. Michelle used the following expression to represent the total number of miles she ran each day last year:
3 × 3 × 3 × 3 × 3 × 3. Write this expression using exponents. How many miles did Michelle run last year?

Choose the letter for the best answer.

5. Cell A divides every 30 minutes. If you start with two cells, how many cells will you have in 3 hours?
   A 6 cells
   B 32 cells
   C 128 cells

6. A soccer team has a phone tree in case a soccer game is cancelled. The coach calls 2 families. Then each family calls 2 other families. How many families will be notified during the 4th round of calls?
   A 2 families
   B 4 families
   C 16 families

7. The Akashi-Kaiko Bridge is the longest suspension bridge in the world. It is about 3^8 feet long. Write the approximate length of the Akashi-Kaiko Bridge in standard form.
   A 6,561 feet
   B 2,187 feet
   C 24 feet

8. The Strahov Stadium is the largest sports stadium in the world. Its capacity is about 12^5 people. Write the capacity of the Strahov Stadium in standard form.
   A 144 people
   B 20,736 people
   C 248,832 people
Problem Solving

Variables and Expressions

Write the correct answer.

1. To cook 4 cups of rice, you use 8 cups of water. To cook 10 cups of rice, you use 20 cups of water. Write an expression to show how many cups of water you should use if you want to cook \( c \) cups of rice. How many cups of water should you use to cook 5 cups of rice?

Solution:

For \( c \) cups of rice:

\[
\frac{4 \text{ cups rice}}{8 \text{ cups water}} = \frac{10}{20} = \frac{1}{2}
\]

\[
\frac{c}{2c} = \frac{1}{2}
\]

You need 2\( c \) cups of water to cook \( c \) cups of rice.

For 5 cups of rice:

\[
\frac{5}{10} = \frac{1}{2}
\]

You need 10 cups of water.

Circle the letter of the correct answer.

2. Sue earns the same amount of money for each hour that she works. In 3 hours, she earns $27. In 8 hours, she earns $72. Write an expression to show how much money Sue earns working \( h \) hours. At this rate, how much money will Sue earn if she works 12 hours?

\[
3h = 27
\]

\[
8h = 72
\]

For \( h \) hours she makes \( \frac{3}{8} \) or \( \frac{9}{24} \) or \( \frac{3}{8} \).

For 12 hours she makes \( \frac{3}{1} \) or \( 3h \).

3. In 2005, 1 United States dollar was worth 28 Russian rubles. How many rubles were equivalent to 10 United States dollars?

\[
1 \text{ US dollar} = 28 \text{ rubles}
\]

\[
10 \text{ US dollars} = ? \text{ rubles}
\]

A 28
B 280
C 2,800

4. In 2005, 1 United States dollar was worth 10 Mexican pesos. How many pesos were equivalent to 5 United States dollars?

\[
1 \text{ US dollar} = 10 \text{ pesos}
\]

\[
5 \text{ US dollars} = ? \text{ pesos}
\]

A 10
B 15
C 50
LESSON 2-1

Practice

Variables and Expressions

Circle the letter of the correct answer.

1. Which of the following is an algebraic expression?
   A. 10 ∙ (3 − 2)
   B. 15 ÷ 5
   C. 9 − n

2. What is the variable in the expression (16 + a) ∙ 5 − 4?
   A. 16
   B. a
   C. n

3. Which of these expressions is a way to rewrite the algebraic expression n + 3?
   A. \( \frac{n}{3} \)
   B. n ∙ 3
   C. \( \frac{3}{n} \)

Evaluate each expression to find the missing values in the tables.

4. Evaluate:

<table>
<thead>
<tr>
<th>n</th>
<th>n + 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

6. If x = 3, what is the value of the expression 6 ÷ x?

5. Evaluate:

<table>
<thead>
<tr>
<th>n</th>
<th>n ∙ 2²</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

7. If x = 2, what is the value of the expression 9 − x?