

Summer Work – Calculus Honors – Mr. Killheffer

Hello Everyone,

I am excited to have you in class this coming school year! I love math and working with students to help them understand and enjoy working with math.

This is the summer work for Calculus Honors. They are practice problems for concepts that I hope you recall from Algebra I and II. These are concepts you need for Calculus.

My expectation for the year is that if you do not know how to do a problem, you do your best to find out how to do it. You have several resources available to you online including khanacademy.org and google.com. If you get stuck, check out an online source.

When school starts, I will spend the first two classes answering any questions you have on the summer work. On the third day of class I will give a test on the summer work that covers the same types of problems to see where you stand.

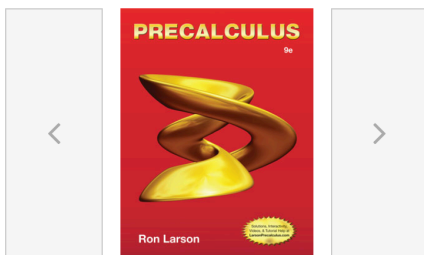
Good luck and I look forward to working with you!

Mr. Killheffer

NOTE: I am assigning odd-numbered problems as usual. Feel free to refer to Calcchat.com to check your work and for assistance if you need it, but don't forget, **"Your goal is not to get it done.... It's to get it."**

This is what it looks like on the calcchat.com opening page when you scroll down and select Precalculus, 9e

Precalculus & College Algebra



- Precalculus with Limits 4e, High School
- College Algebra Real Math Real People 7e
- Algebra and Trigonometry Real Mathematics Real People
- Precalculus 9e**
- Algebra and Trigonometry 9e
- College Algebra 9e
- Precalculus with Limits 2e

Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

1.1 Plotting Points in the Cartesian Plane In Exercises 1 and 2, plot the points in the Cartesian plane.

- (5, 5), (-2, 0), (-3, 6), (-1, -7)
- (0, 6), (8, 1), (5, -4), (-3, -3)

Determining Quadrant(s) for a Point In Exercises 3 and 4, determine the quadrant(s) in which (x, y) is located so that the condition(s) is (are) satisfied.

- $x > 0$ and $y = -2$
- $xy = 4$

5. Plotting, Distance, and Midpoint Plot the points $(-2, 6)$ and $(4, -3)$. Then find the distance between the points and the midpoint of the line segment joining the points.

6. Sales Barnes & Noble had annual sales of \$5.1 billion in 2008 and \$7.0 billion in 2010. Use the Midpoint Formula to estimate the sales in 2009. Assume that the annual sales followed a linear pattern. (Source: Barnes & Noble, Inc.)

1.2 Sketching the Graph of an Equation In Exercises 7–10, construct a table of values. Use the resulting solution points to sketch the graph of the equation.

- $y = 3x - 5$
- $y = -\frac{1}{2}x + 2$
- $y = x^2 - 3x$
- $y = 2x^2 - x - 9$

Finding x - and y -Intercepts In Exercises 11–14, find the x - and y -intercepts of the graph of the equation.

- $y = 2x + 7$
- $y = |x + 1| - 3$
- $y = (x - 3)^2 - 4$
- $y = x\sqrt{4 - x^2}$

Intercepts, Symmetry, and Graphing In Exercises 15–22, identify any intercepts and test for symmetry. Then sketch the graph of the equation.

- $y = -4x + 1$
- $y = 5x - 6$
- $y = 5 - x^2$
- $y = x^2 - 10$
- $y = x^3 + 3$
- $y = -6 - x^3$
- $y = \sqrt{x + 5}$
- $y = |x| + 9$

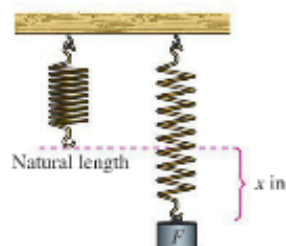
Sketching the Graph of a Circle In Exercises 23–26, find the center and radius of the circle. Then sketch the graph of the circle.

- $x^2 + y^2 = 9$
- $x^2 + y^2 = 4$
- $(x + 2)^2 + y^2 = 16$
- $x^2 + (y - 8)^2 = 81$

27. Writing the Equation of a Circle Write the standard form of the equation of the circle for which the endpoints of a diameter are $(0, 0)$ and $(4, -6)$.

28. Physics The force F (in pounds) required to stretch a spring x inches from its natural length (see figure) is

$$F = \frac{5}{4}x, \quad 0 \leq x \leq 20.$$



(a) Use the model to complete the table.

x	0	4	8	12	16	20
Force, F						

(b) Sketch a graph of the model.

(c) Use the graph to estimate the force necessary to stretch the spring 10 inches.

1.3 Graphing a Linear Equation In Exercises 29–32, find the slope and y -intercept (if possible) of the equation of the line. Sketch the line.

- $y = 3x + 13$
- $y = -10x + 9$
- $y = 6$
- $x = -3$

Finding the Slope of a Line Through Two Points In Exercises 33 and 34, plot the points and find the slope of the line passing through the pair of points.

- $(6, 4)$, $(-3, -4)$
- $(-3, 2)$, $(8, 2)$

Finding an Equation of a Line In Exercises 35 and 36, find an equation of the line that passes through the given point and has the indicated slope m . Sketch the line.

- $(10, -3)$, $m = -\frac{1}{2}$
- $(-8, 5)$, $m = 0$

Finding an Equation of a Line In Exercises 37 and 38, find an equation of the line passing through the points.

- $(-1, 0)$, $(6, 2)$
- $(11, -2)$, $(6, -1)$

Finding Parallel and Perpendicular Lines In Exercises 39 and 40, write the slope-intercept form of the equations of the lines through the given point (a) parallel to and (b) perpendicular to the given line.

- $5x - 4y = 8$, $(3, -2)$
- $2x + 3y = 5$, $(-8, 3)$

41. **Sales** A discount outlet is offering a 20% discount on all items. Write a linear equation giving the sale price S for an item with a list price L .
42. **Hourly Wage** A microchip manufacturer pays its assembly line workers \$12.25 per hour. In addition, workers receive a piecework rate of \$0.75 per unit produced. Write a linear equation for the hourly wage W in terms of the number of units x produced per hour.

1.4 Testing for Functions Represented Algebraically In Exercises 43–46, determine whether the equation represents y as a function of x .

43. $16x - y^4 = 0$ 44. $2x - y - 3 = 0$
 45. $y = \sqrt{1 - x}$ 46. $|y| = x + 2$

Evaluating a Function In Exercises 47 and 48, evaluate the function at each specified value of the independent variable and simplify.

47. $f(x) = x^2 + 1$
 (a) $f(2)$ (b) $f(-4)$ (c) $f(t^2)$ (d) $f(t + 1)$
48. $h(x) = \begin{cases} 2x + 1, & x \leq -1 \\ x^2 + 2, & x > -1 \end{cases}$
 (a) $h(-2)$ (b) $h(-1)$ (c) $h(0)$ (d) $h(2)$

Finding the Domain of a Function In Exercises 49 and 50, find the domain of the function. Verify your result with a graph.

49. $f(x) = \sqrt{25 - x^2}$ 50. $h(x) = \frac{x}{x^2 - x - 6}$

Physics In Exercises 51 and 52, the velocity of a ball projected upward from ground level is given by $v(t) = -32t + 48$, where t is the time in seconds and v is the velocity in feet per second.

51. Find the velocity when $t = 1$.
 52. Find the time when the ball reaches its maximum height. [Hint: Find the time when $v(t) = 0$.]

f Evaluating a Difference Quotient In Exercises 53 and 54, find the difference quotient and simplify your answer.

53. $f(x) = 2x^2 + 3x - 1$, $\frac{f(x+h) - f(x)}{h}$, $h \neq 0$
 54. $f(x) = x^3 - 5x^2 + x$, $\frac{f(x+h) - f(x)}{h}$, $h \neq 0$

1.5 Vertical Line Test for Functions In Exercises 55 and 56, use the Vertical Line Test to determine whether y is a function of x . To print an enlarged copy of the graph, go to MathGraphs.com.

55. $y = (x - 3)^2$ 56. $x = -|4 - y|$

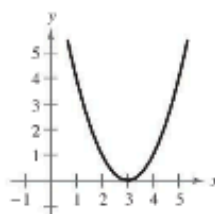


Figure for 55

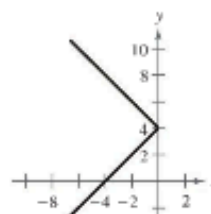


Figure for 56

Finding the Zeros of a Function In Exercises 57–60, find the zeros of the function algebraically.

57. $f(x) = 3x^2 - 16x + 21$ 58. $f(x) = 5x^2 + 4x - 1$
 59. $f(x) = \frac{8x + 3}{11 - x}$ 60. $f(x) = x^3 - x^2$

Describing Function Behavior In Exercises 61 and 62, use a graphing utility to graph the function and visually determine the intervals on which the function is increasing, decreasing, or constant.

61. $f(x) = |x| + |x + 1|$ 62. $f(x) = (x^2 - 4)^2$

Approximating Relative Minima or Maxima In Exercises 63 and 64, use a graphing utility to graph the function and approximate (to two decimal places) any relative minima or maxima.

63. $f(x) = -x^2 + 2x + 1$ 64. $f(x) = x^3 - 4x^2 - 1$

Average Rate of Change of a Function In Exercises 65 and 66, find the average rate of change of the function from x_1 to x_2 .

65. $f(x) = -x^2 + 8x - 4$, $x_1 = 0, x_2 = 4$
 66. $f(x) = 2 - \sqrt{x + 1}$, $x_1 = 3, x_2 = 7$

Even, Odd, or Neither? In Exercises 67–68, determine whether the function is even, odd, or neither. Then describe the symmetry.

67. $f(x) = x^4 - 20x^2$ 68. $f(x) = 2x\sqrt{x^2 + 3}$

1.6 Writing a Linear Function In Exercises 69 and 70, (a) write the linear function f such that it has the indicated function values, and (b) sketch the graph of the function.

69. $f(2) = -6$, $f(-1) = 3$
 70. $f(0) = -5$, $f(4) = -8$

Graphing a Function In Exercises 71–74, sketch the graph of the function.

71. $f(x) = 3 - x^2$ 72. $f(x) = \sqrt{x + 1}$
 73. $g(x) = \frac{1}{x + 5}$
 74. $f(x) = \begin{cases} 5x - 3, & x \geq -1 \\ -4x + 5, & x < -1 \end{cases}$

Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

2.1 Sketching Graphs of Quadratic Functions

In Exercises 1 and 2, sketch the graph of each quadratic function and compare it with the graph of $y = x^2$.

1. (a) $g(x) = -2x^2$ 2. (a) $h(x) = (x - 3)^2$
 (b) $h(x) = x^2 + 2$ (b) $k(x) = \frac{1}{2}x^2 - 1$

Using Standard Form to Graph a Parabola In Exercises 3–8, write the quadratic function in standard form and sketch its graph. Identify the vertex, axis of symmetry, and x -intercept(s).

3. $g(x) = x^2 - 2x$ 4. $f(x) = x^2 + 8x + 10$
 5. $h(x) = 3 + 4x - x^2$ 6. $f(t) = -2t^2 + 4t + 1$
 7. $h(x) = 4x^2 + 4x + 13$ 8. $f(x) = \frac{1}{3}(x^2 + 5x - 4)$

9. Geometry The perimeter of a rectangle is 1000 meters.

- (a) Write the width y as a function of the length x . Use the result to write the area A as a function of x .
 (b) Of all possible rectangles with perimeters of 1000 meters, find the dimensions of the one with the maximum area.

10. Minimum Cost A soft-drink manufacturer has daily production costs of $C = 70,000 - 120x + 0.055x^2$, where C is the total cost (in dollars) and x is the number of units produced. How many units should the manufacturer produce each day to yield a minimum cost?

2.2 Sketching a Transformation of a Monomial Function In Exercises 11 and 12, sketch the graphs of $y = x^n$ and the transformation.

11. $y = x^4$, $f(x) = 6 - x^4$ 12. $y = x^5$, $f(x) = \frac{1}{2}x^5 + 3$

Applying the Leading Coefficient Test In Exercises 13–16, describe the right-hand and left-hand behavior of the graph of the polynomial function.

13. $f(x) = -2x^2 - 5x + 12$ 14. $f(x) = \frac{1}{2}x^3 + 2x$
 15. $g(x) = \frac{1}{4}(x^4 + 3x^2 + 2)$ 16. $h(x) = -x^7 + 8x^2 - 8x$

Sketching the Graph of a Polynomial Function In Exercises 17–20, sketch the graph of the function by (a) applying the Leading Coefficient Test, (b) finding the real zeros of the polynomial, (c) plotting sufficient solution points, and (d) drawing a continuous curve through the points.

17. $g(x) = 2x^3 + 4x^2$
 18. $h(x) = 3x^2 - x^4$
 19. $f(x) = -x^3 + x^2 - 2$
 20. $f(x) = x(x^3 + x^2 - 5x + 3)$



Using the Intermediate Value Theorem In Exercises 21 and 22, (a) use the Intermediate Value Theorem and the *table* feature of a graphing utility to find intervals one unit in length in which the polynomial function is guaranteed to have a zero. (b) Adjust the table to approximate the zeros of the function. Use the *zero* or *root* feature of the graphing utility to verify your results.

21. $f(x) = 3x^3 - x^2 + 3$ 22. $f(x) = x^4 - 5x - 1$

2.3 Long Division of Polynomials In Exercises 23 and 24, use long division to divide.

23. $\frac{30x^2 - 3x + 8}{5x - 3}$ 24. $\frac{5x^3 - 21x^2 - 25x - 4}{x^2 - 5x - 1}$

Using the Factor Theorem In Exercises 25 and 26, use synthetic division to determine whether the given values of x are zeros of the function.

25. $f(x) = 20x^4 + 9x^3 - 14x^2 - 3x$
 (a) $x = -1$ (b) $x = \frac{3}{4}$ (c) $x = 0$ (d) $x = 1$
 26. $f(x) = 3x^3 - 8x^2 - 20x + 16$
 (a) $x = 4$ (b) $x = -4$ (c) $x = \frac{2}{3}$ (d) $x = -1$

Factoring a Polynomial In Exercises 27 and 28, (a) verify the given factor(s) of $f(x)$, (b) find the remaining factors of $f(x)$, (c) use your results to write the complete factorization of $f(x)$, (d) list all real zeros of f , and (e) confirm your results by using a graphing utility to graph the function.

- | Function | Factor(s) |
|---|--------------------|
| 27. $f(x) = 2x^3 + 11x^2 - 21x - 90$ | $(x + 6)$ |
| 28. $f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$ | $(x + 2), (x - 3)$ |

2.4 Writing a Complex Number in Standard Form In Exercises 29 and 30, write the complex number in standard form.

29. $8 + \sqrt{-100}$ 30. $-5i + i^2$

Performing Operations with Complex Numbers In Exercises 31–34, perform the operation and write the result in standard form.

31. $(7 + 5i) + (-4 + 2i)$
 32. $(\sqrt{2} - \sqrt{2}i) - (\sqrt{2} + \sqrt{2}i)$
 33. $7i(11 - 9i)$ 34. $(1 + 6i)(5 - 2i)$

Performing Operations with Complex Numbers In Exercises 35 and 36, perform the operation and write the result in standard form.

35. $\frac{6 + i}{4 - i}$ 36. $\frac{4}{2 - 3i} + \frac{2}{1 + i}$

Complex Solutions of a Quadratic Equation In Exercises 37 and 38, use the Quadratic Formula to solve the quadratic equation.

37. $x^2 - 2x + 10 = 0$ 38. $6x^2 + 3x + 27 = 0$

2.5 Zeros of Polynomial Functions In Exercises 39 and 40, determine the number of zeros of the polynomial function.

39. $g(x) = x^2 - 2x - 8$ 40. $h(t) = t^2 - t^5$

Finding the Zeros of a Polynomial Function In Exercises 41 and 42, find all the zeros of the function.

41. $f(x) = x^3 + 3x^2 - 28x - 60$

42. $f(x) = 4x^4 - 11x^3 + 14x^2 - 6x$

Finding the Zeros of a Polynomial Function In Exercises 43 and 44, write the polynomial as the product of linear factors and list all the zeros of the function.

43. $g(x) = x^3 - 7x^2 + 36$

44. $f(x) = x^4 + 8x^3 + 8x^2 - 72x - 153$

45. Using Descartes's Rule of Signs Use Descartes's Rule of Signs to determine the possible numbers of positive and negative real zeros of $h(x) = -2x^5 + 4x^3 - 2x^2 + 5$.

46. Verifying Upper and Lower Bounds Use synthetic division to verify the upper and lower bounds of the real zeros of $f(x) = 4x^3 - 3x^2 + 4x - 3$.

(a) Upper: $x = 1$ (b) Lower: $x = -\frac{1}{4}$

2.6 Finding Domain and Asymptotes In Exercises 47 and 48, find the domain and the vertical and horizontal asymptotes of the graph of the rational function.

47. $f(x) = \frac{3x}{x + 10}$

48. $f(x) = \frac{8}{x^2 - 10x + 24}$

Sketching the Graph of a Rational Function In Exercises 49–56, (a) state the domain of the function, (b) identify all intercepts, (c) find any asymptotes, and (d) plot additional solution points as needed to sketch the graph of the rational function.

49. $f(x) = \frac{4}{x}$

50. $h(x) = \frac{x - 4}{x - 7}$

51. $f(x) = \frac{x}{x^2 + 1}$

52. $f(x) = \frac{2x^2}{x^2 - 4}$

53. $f(x) = \frac{6x^2 - 11x + 3}{3x^2 - x}$


54. $f(x) = \frac{6x^2 - 7x + 2}{4x^2 - 1}$

55. $f(x) = \frac{2x^3}{x^2 + 1}$

56. $f(x) = \frac{x^2 + 1}{x + 1}$


57. Seizure of Illegal Drugs The cost C (in millions of dollars) for the federal government to seize $p\%$ of an illegal drug as it enters the country is given by

$$C = \frac{528p}{100 - p}, \quad 0 \leq p < 100.$$

-  (a) Use a graphing utility to graph the cost function.
 (b) Find the costs of seizing 25%, 50%, and 75% of the drug.
 (c) According to the model, would it be possible to seize 100% of the drug?

58. Page Design A page that is x inches wide and y inches high contains 30 square inches of print. The top and bottom margins are each 2 inches deep, and the margins on each side are 2 inches wide.

- (a) Write a function for the total area A of the page in terms of x .
 (b) Determine the domain of the function based on the physical constraints of the problem.

 (c) Use a graphing utility to graph the area function and approximate the page size for which the least amount of paper is used.

2.7 Solving an Inequality In Exercises 59–62, solve the inequality. Then graph the solution set.

59. $12x^2 + 5x < 2$

60. $x^3 - 16x \geq 0$

61. $\frac{2}{x + 1} \geq \frac{3}{x - 1}$

62. $\frac{x^2 - 9x + 20}{x} < 0$

63. Population of a Species A biologist introduces 200 ladybugs into a crop field. The population P of the ladybugs can be approximated by the model

$$P = \frac{1000(1 + 3t)}{5 + t}$$

where t is the time in days. Find the time required for the population to increase to at least 2000 ladybugs.

Exploration

64. Writing Describe what is meant by an asymptote of a graph.

True or False? In Exercises 65 and 66, determine whether the statement is true or false. Justify your answer.

65. A fourth-degree polynomial with real coefficients can have -5 , $-8i$, $4i$, and 5 as its zeros.
 66. The domain of a rational function can never be the set of all real numbers.

Next page

Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

3.1 Evaluating an Exponential Function In Exercises 1–6, evaluate the function at the indicated value of x . Round your result to three decimal places.

1. $f(x) = 0.3^x$, $x = 1.5$
2. $f(x) = 30^x$, $x = \sqrt{3}$
3. $f(x) = 2^{-0.5x}$, $x = \pi$
4. $f(x) = 1278^{x/5}$, $x = 1$
5. $f(x) = 7(0.2^x)$, $x = -\sqrt{11}$
6. $f(x) = -14(5^x)$, $x = -0.8$

Transforming the Graph of an Exponential Function In Exercises 7–10, use the graph of f to describe the transformation that yields the graph of g .

7. $f(x) = 5^x$, $g(x) = 5^x + 1$
8. $f(x) = 6^x$, $g(x) = 6^{x+1}$
9. $f(x) = 3^x$, $g(x) = 1 - 3^x$
10. $f(x) = \left(\frac{1}{2}\right)^x$, $g(x) = -\left(\frac{1}{2}\right)^{x+2}$

Graphing an Exponential Function In Exercises 11–16, use a graphing utility to construct a table of values for the function. Then sketch the graph of the function.

11. $f(x) = 4^{-x} + 4$
12. $f(x) = 2.65^{x-1}$
13. $f(x) = 5^{x-2} + 4$
14. $f(x) = 2^{x-6} - 5$
15. $f(x) = \left(\frac{4}{3}\right)^{-x} + 3$
16. $f(x) = \left(\frac{1}{8}\right)^{x+2} - 5$

Using the One-to-One Property In Exercises 17–20, use the One-to-One Property to solve the equation for x .

17. $\left(\frac{1}{3}\right)^{x-3} = 9$
18. $3^{x+3} = \frac{1}{81}$
19. $e^{3x-5} = e^7$
20. $e^{8-2x} = e^{-3}$

Evaluating the Natural Exponential Function In Exercises 21–24, evaluate $f(x) = e^x$ at the indicated value of x . Round your result to three decimal places.

21. $x = 8$
22. $x = \frac{5}{8}$
23. $x = -1.7$
24. $x = 0.278$


Graphing a Natural Exponential Function In Exercises 25–28, use a graphing utility to construct a table of values for the function. Then sketch the graph of the function.

25. $h(x) = e^{-x/2}$
26. $h(x) = 2 - e^{-x/2}$
27. $f(x) = e^{x+2}$
28. $s(t) = 4e^{-2/t}$, $t > 0$

29. Waiting Times The average time between incoming calls at a switchboard is 3 minutes. The probability F of waiting less than t minutes until the next incoming call is approximated by the model $F(t) = 1 - e^{-t/3}$. The switchboard has just received a call. Find the probability that the next call will be within

- (a) $\frac{1}{2}$ minute. (b) 2 minutes. (c) 5 minutes.

30. Depreciation After t years, the value V of a car that originally cost \$23,970 is given by $V(t) = 23,970\left(\frac{3}{4}\right)^t$.

-  (a) Use a graphing utility to graph the function.
- (b) Find the value of the car 2 years after it was purchased.
- (c) According to the model, when does the car depreciate most rapidly? Is this realistic? Explain.
- (d) According to the model, when will the car have no value?

Compound Interest In Exercises 31 and 32, complete the table to determine the balance A for P dollars invested at rate r for t years and compounded n times per year.

n	1	2	4	12	365	Continuous
A						

31. $P = \$5000$, $r = 3\%$, $t = 10$ years
32. $P = \$4500$, $r = 2.5\%$, $t = 30$ years

3.2 Writing a Logarithmic Equation In Exercises 33–36, write the exponential equation in logarithmic form. For example, the logarithmic form of $2^3 = 8$ is $\log_2 8 = 3$.

33. $3^3 = 27$
34. $25^{3/2} = 125$
35. $e^{0.8} = 2.2255 \dots$
36. $e^0 = 1$

Evaluating a Logarithmic Function In Exercises 37–40, evaluate the function at the indicated value of x without using a calculator.

37. $f(x) = \log x$, $x = 1000$
38. $g(x) = \log_9 x$, $x = 3$
39. $g(x) = \log_2 x$, $x = \frac{1}{4}$
40. $f(x) = \log_3 x$, $x = \frac{1}{81}$

Using the One-to-One Property In Exercises 41–44, use the One-to-One Property to solve the equation for x .

41. $\log_4(x + 7) = \log_4 14$
42. $\log_8(3x - 10) = \log_8 5$
43. $\ln(x + 9) = \ln 4$
44. $\ln(2x - 1) = \ln 11$

Sketching the Graph of a Logarithmic Function In Exercises 45–48, find the domain, x -intercept, and vertical asymptote of the logarithmic function and sketch its graph.

45. $g(x) = \log_7 x$
46. $f(x) = \log\left(\frac{x}{3}\right)$
47. $f(x) = 4 - \log(x + 5)$
48. $f(x) = \log(x - 3) + 1$

Evaluating a Logarithmic Function on a Calculator In Exercises 49–52, use a calculator to evaluate the function at the indicated value of x . Round your result to three decimal places, if necessary.

49. $f(x) = \ln x$, $x = 22.6$ 50. $f(x) = \ln x$, $x = e^{-12}$
 51. $f(x) = \frac{1}{2} \ln x$, $x = \sqrt{e}$
 52. $f(x) = 5 \ln x$, $x = 0.98$

Graphing a Natural Logarithmic Function In Exercises 53–56, find the domain, x -intercept, and vertical asymptote of the logarithmic function and sketch its graph.

53. $f(x) = \ln x + 3$ 54. $f(x) = \ln(x - 3)$
 55. $h(x) = \ln(x^2)$ 56. $f(x) = \frac{1}{4} \ln x$
 57. **Antler Spread** The antler spread a (in inches) and shoulder height h (in inches) of an adult male American elk are related by the model

$$h = 116 \log(a + 40) - 176.$$

Approximate the shoulder height of a male American elk with an antler spread of 55 inches.

58. **Snow Removal** The number of miles s of roads cleared of snow is approximated by the model

$$s = 25 - \frac{13 \ln(h/12)}{\ln 3}, \quad 2 \leq h \leq 15$$

where h is the depth of the snow in inches. Use this model to find s when $h = 10$ inches.

3.3 Using the Change-of-Base Formula In Exercises 59–62, evaluate the logarithm using the change-of-base formula (a) with common logarithms and (b) with natural logarithms. Round your results to three decimal places.

59. $\log_2 6$ 60. $\log_{12} 200$
 61. $\log_{1/2} 5$ 62. $\log_3 0.28$

Using Properties of Logarithms In Exercises 63–66, use the properties of logarithms to rewrite and simplify the logarithmic expression.

63. $\log 18$ 64. $\log_2 \left(\frac{1}{12}\right)$
 65. $\ln 20$ 66. $\ln(3e^{-4})$

Expanding a Logarithmic Expression In Exercises 67–72, use the properties of logarithms to expand the expression as a sum, difference, and/or constant multiple of logarithms. (Assume all variables are positive.)

67. $\log_5 5x^2$ 68. $\log 7x^4$
 69. $\log_3 \frac{9}{\sqrt{x}}$ 70. $\log_7 \frac{\sqrt[3]{x}}{14}$
 71. $\ln x^2 y^2 z$ 72. $\ln \left(\frac{y-1}{4}\right)^2$, $y > 1$

Condensing a Logarithmic Expression In Exercises 73–78, condense the expression to the logarithm of a single quantity.

73. $\log_2 5 + \log_2 x$
 74. $\log_6 y - 2 \log_6 z$
 75. $\ln x - \frac{1}{4} \ln y$
 76. $3 \ln x + 2 \ln(x + 1)$
 77. $\frac{1}{2} \log_3 x - 2 \log_3(y + 8)$
 78. $5 \ln(x - 2) - \ln(x + 2) - 3 \ln x$

79. **Climb Rate** The time t (in minutes) for a small plane to climb to an altitude of h feet is modeled by $t = 50 \log[18,000/(18,000 - h)]$, where 18,000 feet is the plane's absolute ceiling.

(a) Determine the domain of the function in the context of the problem.

-  (b) Use a graphing utility to graph the function and identify any asymptotes.

(c) As the plane approaches its absolute ceiling, what can be said about the time required to increase its altitude?

(d) Find the time for the plane to climb to an altitude of 4000 feet.

80. **Human Memory Model** Students in a learning theory study took an exam and then retested monthly for 6 months with an equivalent exam. The data obtained in the study are given by the ordered pairs (t, s) , where t is the time in months after the initial exam and s is the average score for the class. Use the data to find a logarithmic equation that relates t and s .


- (1, 84.2), (2, 78.4), (3, 72.1),
 (4, 68.5), (5, 67.1), (6, 65.3)

3.4 Solving a Simple Equation In Exercises 81–86, solve for x .

81. $5^x = 125$ 82. $6^x = \frac{1}{216}$
 83. $e^x = 3$ 84. $\log_6 x = -1$
 85. $\ln x = 4$ 86. $\ln x = -1.6$

Solving an Exponential Equation In Exercises 87–90, solve the exponential equation algebraically. Approximate the result to three decimal places.

87. $e^{4x} = e^{x^2+3}$ 88. $e^{3x} = 25$
 89. $2^x - 3 = 29$ 90. $e^{2x} - 6e^x + 8 = 0$

 **Graphing and Solving an Exponential Equation** In Exercises 91 and 92, use a graphing utility to graph and solve the equation. Approximate the result to three decimal places. Verify your result algebraically.

91. $25e^{-0.3x} = 12$
 92. $2^x = 3 + x - e^x$